An Overview of Effect Sizes and Meta-analysis

KTDRR Research Evidence Training

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Introduction

- Ryan Williams
  - Principal Researcher, American Institutes for Research (AIR)
  - Principal Investigator: Institute of Education Sciences (IES)-funded meta-analysis focusing on heterogeneity in mathematics intervention effects
  - Co-Principal Investigator: IES-funded training grant on meta-analysis
- Josh Polanin
  - Principal Researcher, AIR
  - Principal Investigator: two National Institute of Justice (NIJ)-funded reviews on school violence
  - Co-Principal Investigator: IES training grant on meta-analysis; IES-funded review on college aid
  - Project Director: What Works Clearinghouse Statistics, Website, and Training (SWAT) contract

Overview

- Effect Size
  - What is an effect size, and why is it important in meta-analysis?
  - How to calculate an effect size depending on what the underlying data are
  - Three varieties: standardized mean-difference, odds ratio, correlation
- Meta-analysis
  - Why synthesize effect sizes?
  - Basics of fixed- and random-effects models
  - Assessing heterogeneity among effect sizes
  - Explaining heterogeneity based on units, treatments, outcomes, and settings (UTOS)
- If time allows—questions at the end

What is an effect size, and why is it important in meta-analysis?

- Start with a thought experiment: Imagine you are interested in understanding whether various programs—all using the same basic tenets—have an impact on an outcome domain.
  - Example: *Therapeutic Effects of Horseback Riding Interventions: A Systematic Review and Meta-analysis* (Stergiou, Tzoufi, Ntzani, Varvarousis, Beris, & Ploumis, 2017)
  - "The purpose of this review was to determine whether therapeutic riding and hippotherapy improve *balance*…" (among many other outcomes)
- How *balance* is measured across each included study probably varies—but each measure focuses on the same underlying construct.

What is an effect size, and why is it important in meta-analysis? (Cont.)

- Meta-analysis expresses the results of each study using a quantitative index of effect size.
- Effect sizes are measures of the strength or magnitude of a relationship of interest.
- Effect sizes have the advantage of being comparable (i.e., they estimate the same thing) across all the studies and therefore can be summarized across studies in the meta-analysis.

What effect size should be calculated?

- Effect sizes can be expressed in many different metrics.
  - *d*, *r*, *OR*, *RR*, etc.
    - The decision about which metric to use is based on what the primary authors report.
- Effect sizes can be unstandardized or standardized.
  - Unstandardized = expressed in measurement units (do not have the properties we need!)
  - Standardized = expressed in standardized measurement units
- Effect sizes should be accompanied by their standard errors; these get calculated along with the effect size and will be used in meta-analytic calculations.

The *d* family

- The standardized mean difference.
- Used when we are interested in two-group comparisons using means.
- Groups could be two experimental groups or, in an observational study, two groups of interest, such as boys versus girls.

The *d* family (Cont.)

- Notation:

  Group means: $\bar{X}_{G1}, \bar{X}_{G2}$

  Group sample sizes: n_{G1}, n_{G2}

  Total sample size: $N = n_{G1} + n_{G2}$

  Group standard deviations: s_{G1}, s_{G2}

The *d* family (Cont.)

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$$ES_{sm} = \frac{\bar{X}_{G1} - \bar{X}_{G2}}{s_p}$$

Pooled sample standard deviation (i.e., a weighted average standard deviation)

The *d* family (Cont.)

- The regular effect size formula is biased, especially with small samples.
- We can easily apply a correction for this bias:

$$ES'_{sm} = \left[1 - \frac{3}{4N - 9}\right] ES_{sm}$$

- This produces a Hedges' g effect size, or bias-corrected estimate.

The *d* family (Cont.)

- Each effect size needs a measure of precision:

$$SE_{sm} = \sqrt{\frac{n_{G1} + n_{G2}}{n_{G1} n_{G2}} + \frac{{ES'_{sm}}^2}{2(n_{G1} + n_{G2})}}$$

The *d* family (Cont.)

MULTISYSTEMIC TREATMENT 957

Table 1
Group Means, Standard Deviations, and F Values for Outcome Measures

Measure	Multisystemic therapy Pre	Multisystemic therapy Post	Usual services Pre	Usual services Post	*F*	*p*
Ultimate outcomes						
Arrests[a]					3.94	.050
M	—	0.87	—	1.52		
SD	—	1.34	—	1.55		
Incarceration[a] (in weeks)					7.77	.006
M	—	5.8	—	16.2		
SD	—	13.9	—	19.1		
SRD					4.14	.047
M	11.5	2.9	12.9	8.6		
SD	15.7	5.1	14.3	16.5		

Source: Henggeler, S. W., Melton, G. B. & Smith, L. A. (1992). Family preservation sing multisystemic therapy: An effective alternative to incarcerating serious juvenile offenders. *Journal of Consulting and Clinical Psychology, 60*(6), *953–961.*

The *d* family (Cont.)

- We can take the values reported in Table 1 and compute an effect size and its standard error:

$$s_p = \sqrt{\frac{(43-1)13.9^2 + (41-1)19.1^2}{(43-1)+(41-1)}} = 16.6$$

$$ES_{sm} = \frac{5.8 - 16.2}{16.6} = -.63$$

$$ES'_{sm} = \left[1 - \frac{3}{4(43+41)-9}\right] - .63 = -.62$$

$$SE_{sm} = \sqrt{\frac{43+41}{43 \times 41} + \frac{-.62^2}{2(43+41)}} = .22$$

The *r* family

- The correlation coefficient, or *r* family effects, may be appropriate when …
  - studies have a continuous outcome measure,
  - study designs assess the relation between a quantitative predictor and the outcome (possibly controlling for covariates), or
  - the analysis uses regression (or the general linear model).

The *r* family (Cont.)

- When using the correlation, you (or a computer program) will do these two things:
  1. Translate it into Fisher's z:

$$Z_r = \frac{1}{2} \ln\left(\frac{1 + r}{1 - r}\right)$$

  2. Calculate the standard error of Z:

$$SE_Z = \frac{1}{\sqrt{n - 3}}$$

The *r* family (Cont.)

- Practically—the formulas are important to know, but you probably won't use them directly.

- Instead—if the correlation is of interest, you will need to locate it in the study and locate its sample size.

The odds ratio family(ish)

- Consider a study in which a treatment group (Tx) and a control group (Cx) are compared with respect to the frequency of a binary characteristic among the participants.
- In each group, we will count how many participants satisfy the binary outcome of interest (e.g., passing a test, graduating, being cured of a disease, etc.).
- The odds ratio is one of a few effect sizes that can be calculated in these scenarios (but it is not the only one).

The odds ratio family(ish) (Cont.)

Study Group	Success	Failure	TOTAL
Treatment	5	14	19
Comparison	6	12	18
TOTAL	11	26	37

Odds of treatment "success" $\frac{5}{14} = .36$

Odds of comparison "success" $\frac{6}{12} = .50$

The "odds ratio" is literally just that—a ratio of two odds—in this case, the odds of treatment success divided by the odds of comparison group success.

$$\frac{.36}{.50} = .72$$

The odds of success in treatment are .72 times the odds of success in control.

The odds ratio family(ish) (Cont.)

- When using the correlation, you (or a computer program) will do these two things:
  1. Transform it to the log odds ratio:

$$Log\ odds\ ratio = \ln(OR)$$

  2. Calculate the log odds ratio standard error:

$$SE_{\ln(OR)} = \sqrt{\frac{1}{Cell\ "a"} + \frac{1}{Cell\ "b"} + \frac{1}{Cell\ "c"} + \frac{1}{Cell\ "d"}}$$

	S	F
Tr	a	b
Co	c	d

Meta-analysis Introduction

- Computing an average effect is done in two general ways:
  - Fixed-effects model
  - Random-effects model
- Both approaches typically will use an estimate of precision to calculate a weighted mean effect size.
- BUT, the two approaches differ in how they characterize those weights.

Meta-analysis Introduction (Cont.)

- Computing an average effect is done in two general ways:
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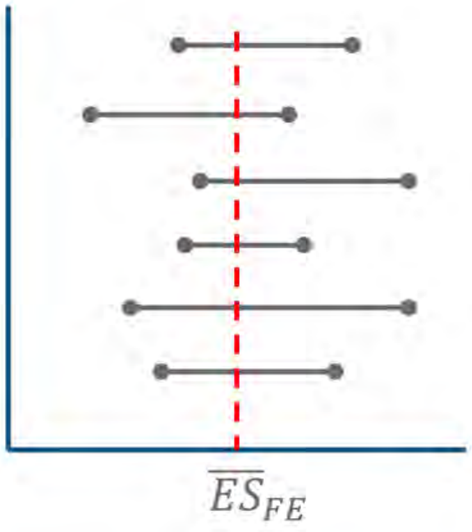
Meta-analysis Slides

- The fixed-effects model considers ***one*** of variation: sampling variance.



$$\overline{ES}_{FE} = \frac{\sum_{i=1}^{j}(w_i ES_i)}{\sum_{i=1}^{j}(w_i)}$$

$$SE_{FE} = \sqrt{\frac{1}{\sum_{i=1}^{j} w_i}}$$

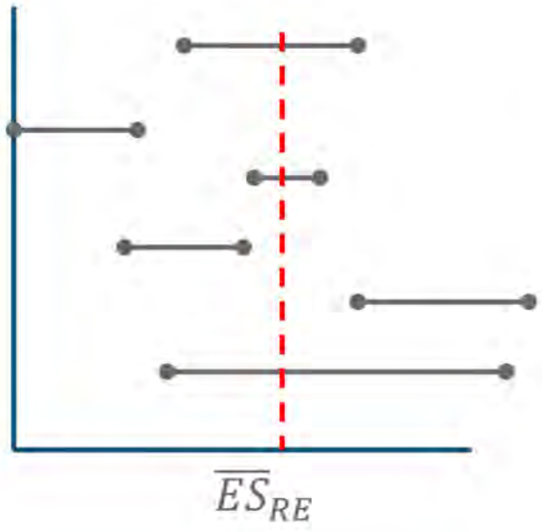
$$w_i = \frac{1}{SE_i^2}$$

Meta-analysis Slides (Cont.)

- This model is helpful when the effect sizes are homogeneous.
- That is, the only reason they are identical is because they have different samples.
- This is a strong assumption.

Meta-analysis Slides (Cont.)

- The random-effects model considers ***two*** sources of variation: sampling variance and between-study variance.



$$\overline{ES}_{RE} = \frac{\Sigma_{i=1}^{j}(w_i ES_i)}{\Sigma_{i=1}^{j}(w_i)}$$

$$SE_{RE} = \sqrt{\frac{1}{\Sigma_{i=1}^{j} w_i}}$$

$$w_i = \frac{1}{SE_i^2 + \hat{\tau}^2}$$

Meta-analysis Slides (Cont.)

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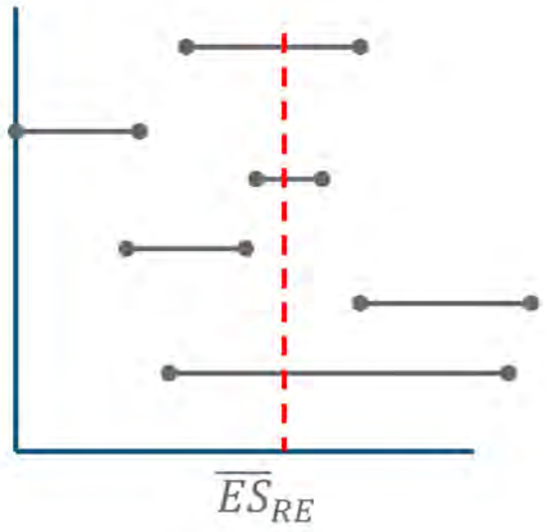


$$\overline{ES}_{RE} = \frac{\sum_{i=1}^{j}(w_i ES_i)}{\sum_{i=1}^{j}(w_i)} \qquad SE_{RE} = \sqrt{\frac{1}{\sum_{i=1}^{j} w_i}}$$

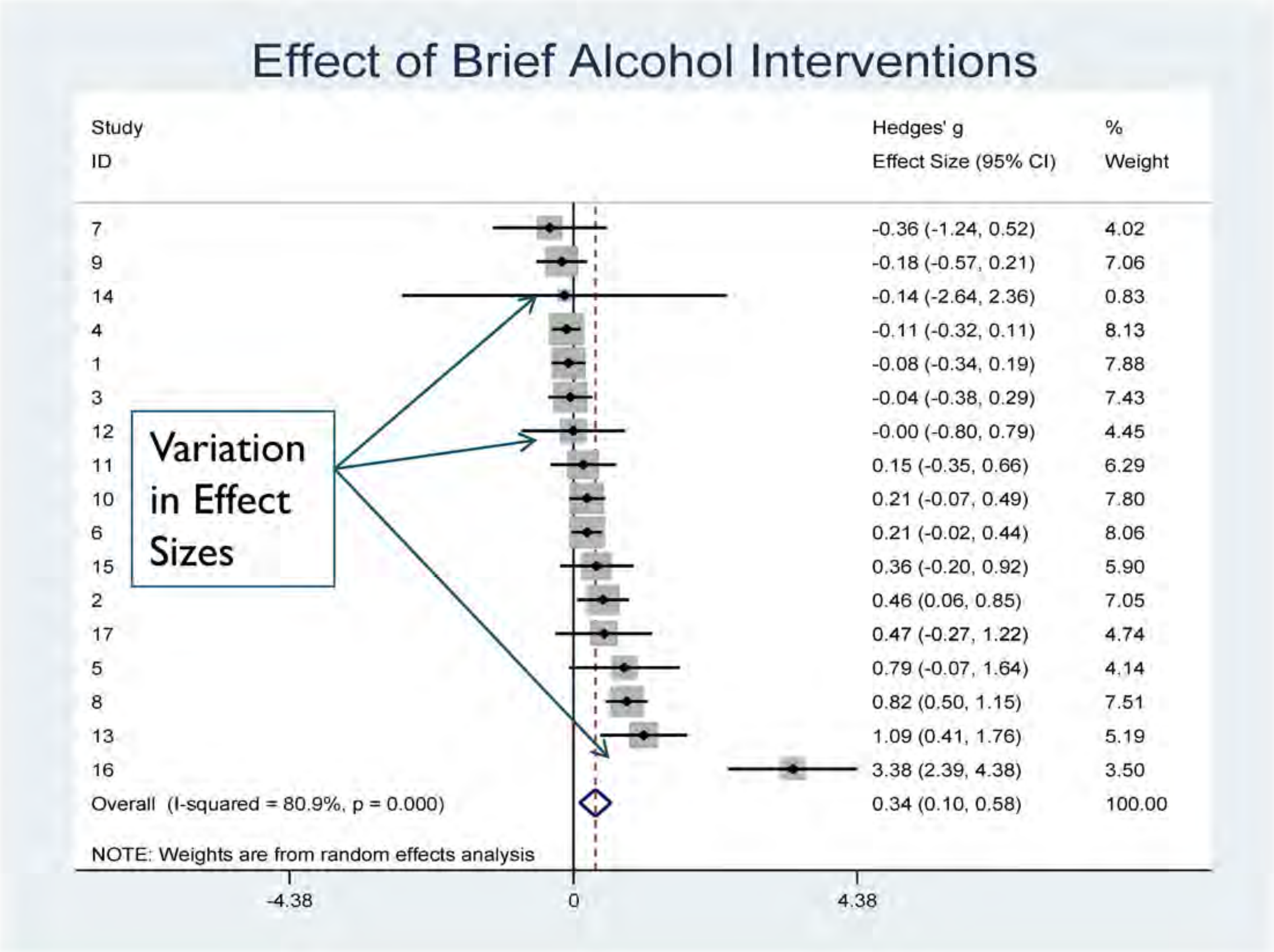
$$w_i = \frac{1}{SE_i^2 + \hat{\tau}^2}$$

Estimated heterogeneity

Meta-analysis Slides (Cont.)

- Because the random-effects model uses two sources of variation, the standard error around the mean effect will be larger than the standard error for an equally sized fixed-effects model.
- BUT, unless the multiple effect sizes are coming from maximum-control conditions (i.e., carefully scripted, laboratory-type replications), it is hard to rule out a random-effects model.
- For this reason, we strongly encourage using a random-effects model. If your data are statistically homogeneous, it won’t hurt you!

What is effect size heterogeneity?



Source: Hennessy & Tanner-Smith, *Prevention Science*, 2015

Quantifying heterogeneity

- Three primary statistics:
  - Q – Tells us if the variation is different from chance.
  - τ^2 – Tells us the magnitude of the variation.
  - I^2 – Tells us the proportion of true variation among effects (taking into account the possibility of random variation).

- These allow us to turn visual information into quantitative information.

Explaining heterogeneity

- What to do with sufficient heterogeneity?

- One-way moderator analyses (old approach)
- One-variable meta-regression (old approach)

- Multiple variable meta-regression (best approach)
  - Control for confounding factors
  - Reduce type 1 errors
  - Easier to interpret

Thank you!

Please take a few minutes to respond to the brief Evaluation Survey:

https://www.surveygizmo.com/s3/4552615/Webcast-Eval-Effect-Sizes-Meta-Analysis

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