Meta-analysis of Single-Case Experimental Designs

An Overview of Current Synthesis Methods

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Acknowledgments

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1. Through support from the Institute of Education Sciences (IES), U.S. Department of Education, I was able to study single-case experimental design (SCD) meta-analysis approaches over the last decade.

   The opinions expressed herein are those of the authors and do not represent the views of IES or the U.S. Department of Education.

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   - R305D150007, Multilevel modeling of single-subject experimental data: Handling data and design complexities
   - R305D190022, Assessing generalizability and variability of single-case design effect sizes using multilevel modeling, including moderators

2. Dr. James Pustejovsky, whose valuable collaboration on slide content created for the IES SCD research institute, informed the slides developed for this presentation.

Dr. Mariola Moeyaert

Agenda

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1. Introduction
2. Approaches to Meta-analysis of Single-Case Experimental Designs
   - 2.1 Meta-analysis of Study-Level Summary Effect Sizes
   - 2.2 Meta-analysis of Case-Level Effect Sizes
   - 2.3 Raw Data Meta-analysis
3. Concluding Thoughts
4. Questions and Answers

1. Introduction

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Single-case experimental designs (SCDs) are

- designed experiments in which one unit is observed repeatedly during a certain period of time under different levels of at least one manipulated variable.
- experimental designs with the potential to demonstrate a causal effect.

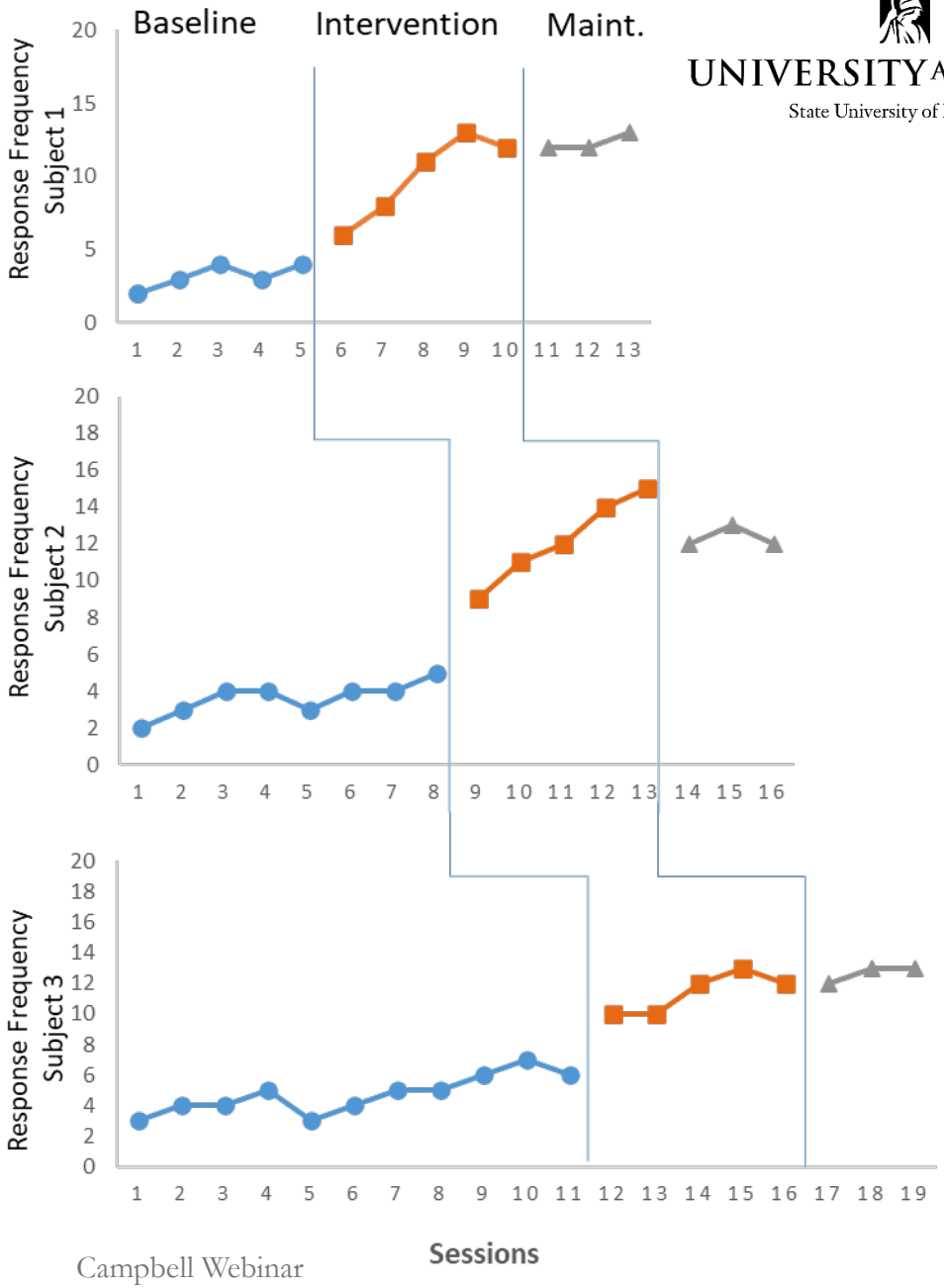
The SCD logic has many variations, but all SCDs often involve repeated, systematic measurement of a dependent variable before, during, and/or after the active manipulation of an independent variable.

1. Introduction

SCD example:

Multiple baseline design –

Replication across subjects



1. Introduction

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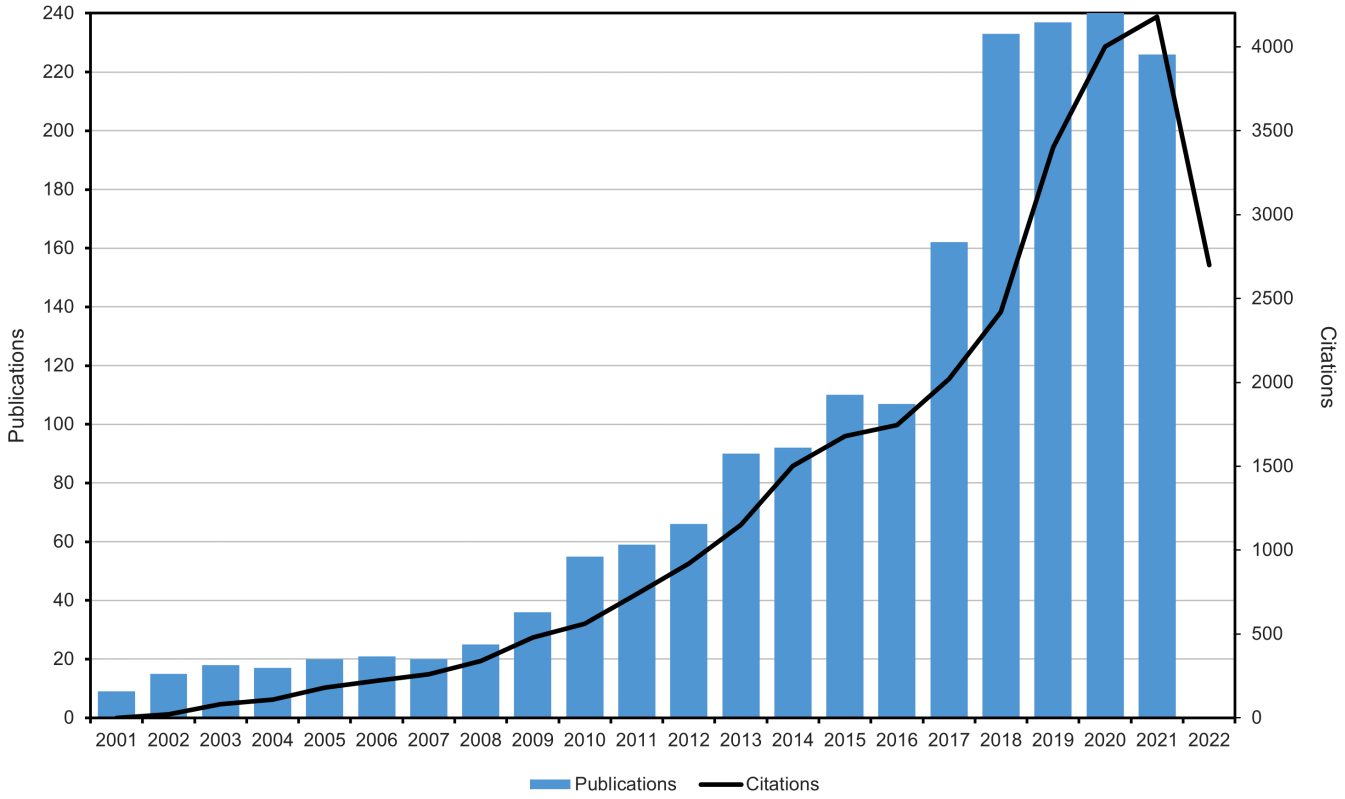
Exponential increase in popularity



Source: Web of Science, Keywords: TS = (single-case* OR single-subject* OR interrupted time series* of intra-subject* or n-of-1*); field = education.

1. Introduction

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Evidence-Based Education Policy

- We have entered an era in which scientific evidence will increasingly inform policy.
- Combining evidence from multiple SCD studies, using **meta-analytic techniques**, can provide a basis for generalization about effects of intervention.

- Using **meta-analysis**, the focus is on
  - ***Summarizing magnitude*** of intervention effects.
  - Investigating ***intervention heterogeneity***.
  - Identifying ***moderators*** to explain intervention heterogeneity.

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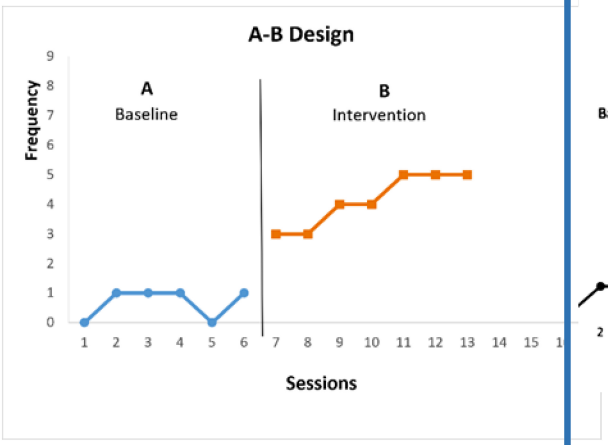
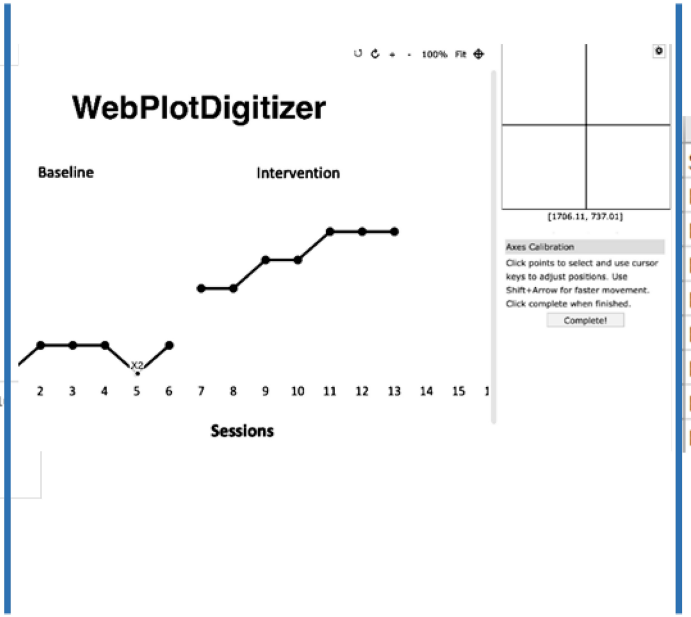
Before we can get started:

- Raw SCD data from all participant graphs need to be retrieved.
- Data retrieval software programs can be used for this purpose (e.g., Moeyaert et al., 2016; WebPlotDigitizer).

Participant graph → Raw data retrieval → Dataset





A	B	C	D	E
Study	Case	Phase	Time	Y
Bambara1995	Al	0	1	3
Bambara1995	Al	0	2	3
Bambara1995	Al	0	3	2
Bambara1995	Al	0	4	4
Bambara1995	Al	1	5	1
Bambara1995	Al	1	6	1
Bambara1995	Al	1	7	0
Bambara1995	Al	1	8	0

2. Approaches to SCD Meta-analysis

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Three broad approaches to meta-analysis:

2.1 Meta-analysis of study-level summary effect sizes

2.2 Meta-analysis of case-level effect sizes

2.3 Raw data meta-analysis (i.e., individual patient data meta-analysis)

These approaches differ in multiple respects:

- Level of analysis
- Available effect size metrics
- Modeling assumptions

2. Approaches to SCD Meta-analysis

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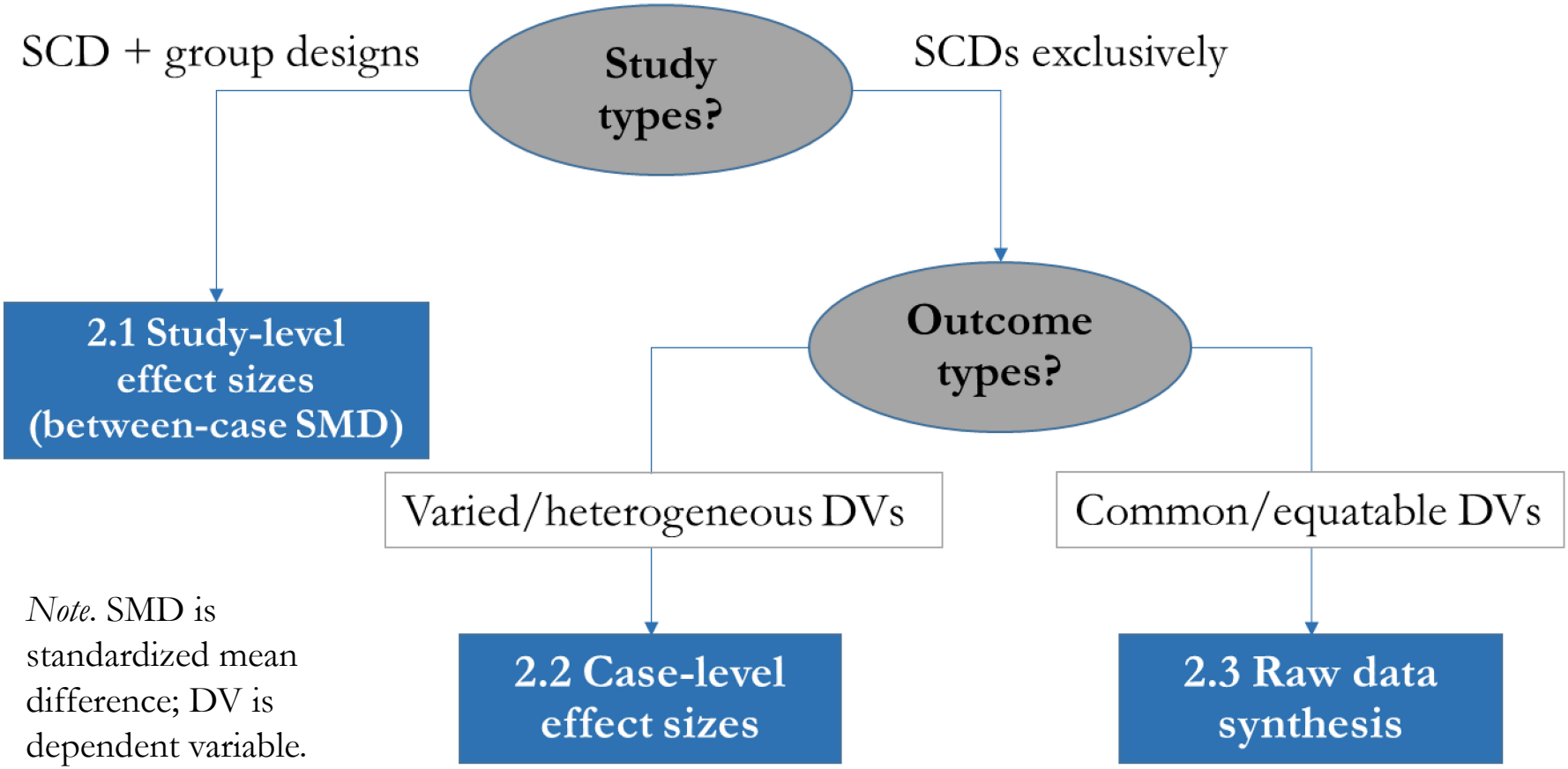
Three broad approaches to meta-analysis:



Note. SMD is standardized mean difference; DV is dependent variable.

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Data to illustrate the three broad approaches:

- Inclusion criteria:
  - Design needs to be an SCD.
  - Population is K–12 students with identified disability.
  - A graphic organizer intervention was implemented.
  - Outcome is reading comprehension.
  - The effect of the intervention on reading comprehension was reported graphically, with a clearly identifiable baseline phase and an intervention phase.

- A total of 23 SCD studies met the inclusion criteria.

- For this illustration, a random selection of 10 SCD studies was sampled.

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Illustration of three broad approaches to meta-analysis:

- Raw meta-analytic dataset

	A	B	C	D	E	F	G	I	J	N	O	P	S	T	U	V	W	X	Y
1	Study_Name	Study	Case	Session	Outcomes	Time	D	Design	Type_Outcome	Interventionist	Digital	sability_1	Quality	_integrity	Dosage	Social_Validity	Age	Grade	Gender
2	1.Alves et al., 2015	1	1	0.937	41.822	1	0	1	1	1	0	1	1	1	10	0	10	5	0
3	1.Alves et al., 2015	1	1	1.937	41.822	2	0	1	1	1	0	1	1	1	10	0	10	5	0
4	1.Alves et al., 2015	1	1	2.988	-0.428	3	0	1	1	1	0	1	1	1	10	0	10	5	0
5	1.Alves et al., 2015	1	1	3.887	83.555	4	1	1	1	1	0	1	1	1	10	0	10	5	0
6	1.Alves et al., 2015	1	1	4.899	83.208	5	1	1	1	1	0	1	1	1	10	0	10	5	0
7	1.Alves et al., 2015	1	1	5.912	83.033	6	1	1	1	1	0	1	1	1	10	0	10	5	0
8	1.Alves et al., 2015	1	1	6.851	93.389	7	1	1	1	1	0	1	1	1	10	0	10	5	0
9	1.Alves et al., 2015	1	1	7.815	103.395	8	1	1	1	1	0	1	1	1	10	0	10	5	0
10	1.Alves et al., 2015	1	1	8.839	93.046	9	1	1	1	1	0	1	1	1	10	0	10	5	0
11	1.Alves et al., 2015	1	1	9.864	92.697	10	1	1	1	1	0	1	1	1	10	0	10	5	0
12	1.Alves et al., 2015	1	1	10.839	103.046	11	1	1	1	1	0	1	1	1	10	0	10	5	0
13	1.Alves et al., 2015	1	1	11.840	92.701	12	1	1	1	1	0	1	1	1	10	0	10	5	0
14	1.Alves et al., 2015	1	1	12.816	92.360	13	1	1	1	1	0	1	1	1	10	0	10	5	0
15	1.Alves et al., 2015	1	1	14.879	49.763	14	3	1	1	1	0	1	1	1	10	0	10	5	0
16	1.Alves et al., 2015	1	2	0.934	52.692	1	0	1	1	1	0	5	1	1	10	0	11	5	1
17	1.Alves et al., 2015	1	2	1.948	31.731	2	0	1	1	1	0	5	1	1	10	0	11	5	1
18	1.Alves et al., 2015	1	2	2.910	42.308	3	0	1	1	1	0	5	1	1	10	0	11	5	1
19	1.Alves et al., 2015	1	2	3.857	84.615	4	1	1	1	1	0	5	1	1	10	0	11	5	1
20	1.Alves et al., 2015	1	2	4.859	52.692	5	1	1	1	1	0	5	1	1	10	0	11	5	1
21	1.Alves et al., 2015	1	2	5.807	84.615	6	1	1	1	1	0	5	1	1	10	0	11	5	1
22	1.Alves et al., 2015	1	2	6.808	63.269	7	1	1	1	1	0	5	1	1	10	0	11	5	1
23	1.Alves et al., 2015	1	2	8.758	73.846	8	1	1	1	1	0	5	1	1	10	0	11	5	1
24	1.Alves et al., 2015	1	2	9.744	84.615	9	1	1	1	1	0	5	1	1	10	0	11	5	1
25	1.Alves et al., 2015	1	2	10.706	95.385	10	1	1	1	1	0	5	1	1	10	0	11	5	1
26	1.Alves et al., 2015	1	2	11.668	105.385	11	1	1	1	1	0	5	1	1	10	0	11	5	1
27	1.Alves et al., 2015	1	2	12.669	95.192	12	1	1	1	1	0	5	1	1	10	0	11	5	1
28	1.Alves et al., 2015	1	2	14.631	105.385	13	3	1	1	1	0	5	1	1	10	0	11	5	1
29	1.Alves et al., 2015	1	3	1.000	70.702	1	0	1	1	1	0	9	1	1	10	0	9	3	0
30	1.Alves et al., 2015	1	3	2.000	70.702	2	0	1	1	1	0	9	1	1	10	0	9	3	0
31	1.Alves et al., 2015	1	3	3.000	20.175	3	0	1	1	1	0	9	1	1	10	0	9	3	0
32	1.Alves et al., 2015	1	3	4.015	80.526	4	0	1	1	1	0	9	1	1	10	0	9	3	0
33	1.Alves et al., 2015	1	3	5.000	40.175	5	0	1	1	1	0	9	1	1	10	0	9	3	0
34	1.Alves et al., 2015	1	3	5.925	80.702	6	0	1	1	1	0	9	1	1	10	0	9	3	0

2. Approaches to SCD Meta-analysis

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2.1 Meta-analysis of Study-Level Effect Sizes

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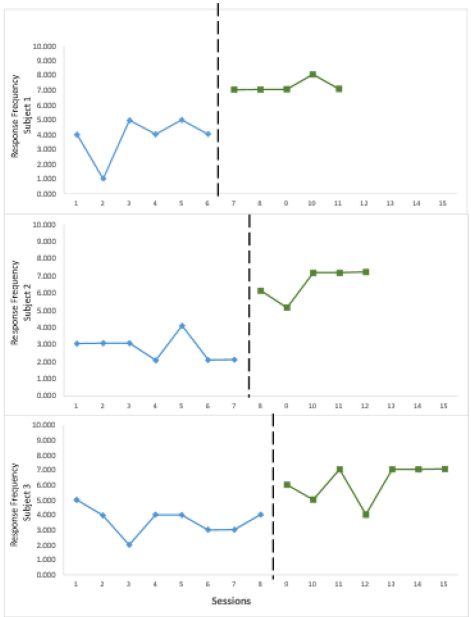
2.1 Meta-analysis of Study-Level Effect Sizes

Effect size: Between-case standardized mean difference (a.k.a. design-comparable effect size)

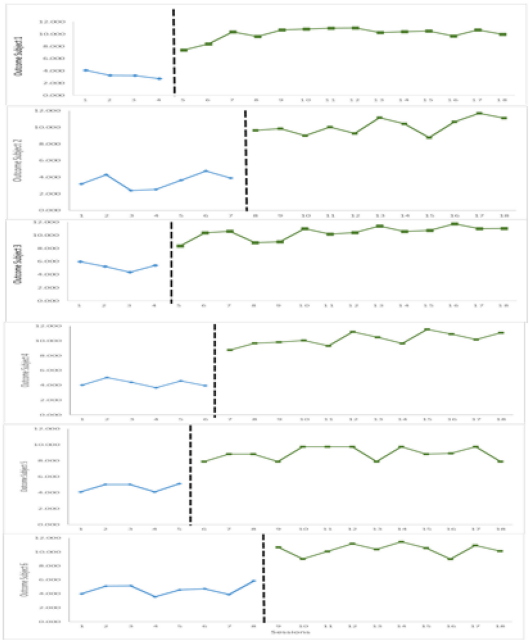
Single-number summary of average study intervention effect

3. Bethune & Wood, 2013



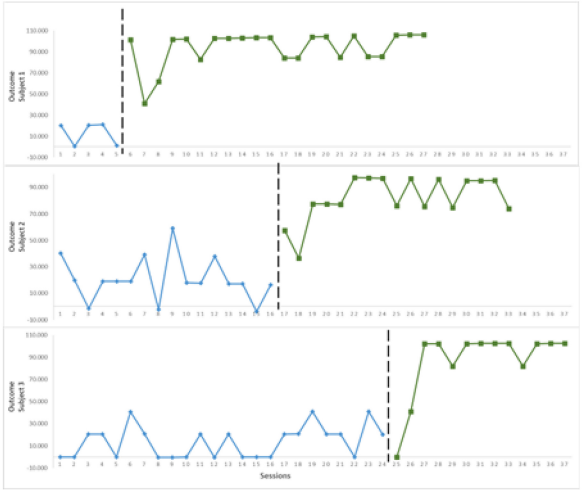
g_3, SE_3

24. Grünke et al., 2013



g_{24}, SE_{24} ...

45. Stringfield et al., 2011



g_{45}, SE_{45}

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2.1 Meta-analysis of Study-Level Effect Sizes

Goal: Summarize study-level average ES (effect size) in a metric that is theoretically comparable to ES from a between-group design.

- Can then use conventional meta-analysis methods for synthesis.

Available **ES metric**: Between-case SMD (standardized mean difference), also called D-CES (design-comparable effect size).

- scdhlm web app and R package (https://www.jepusto.com/software/scdhlm/)
- Between-case SMD ES for SCDs: a primer and tutorial using the scdhlm web application (https://onlinelibrary.wiley.com/doi/10.4073/cmdp.2016.1)

Modeling **assumptions**:

- Hierarchical model for data from each study
- Treatment initiation and follow-up times in hypothetical group design

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2.1 Meta-analysis of Study-Level Effect Sizes

- What is the SMD from a between-groups experiment?

$$\delta_{BC} = \frac{\begin{pmatrix}\text{Average outcome if}\\ \text{everybody gets treatment}\end{pmatrix} - \begin{pmatrix}\text{Average outcome if}\\ \text{nobody gets treatment}\end{pmatrix}}{\begin{pmatrix}\text{Outcome standard dev.}\\ \text{nobody gets treatment}\end{pmatrix}}$$

$$\delta_{BC} = \frac{\begin{pmatrix}\text{Average outcome if}\\ \text{everybody gets treatment}\end{pmatrix} - \begin{pmatrix}\text{Average outcome if}\\ \text{nobody gets treatment}\end{pmatrix}}{\sqrt{\begin{pmatrix}\text{Within}-\text{participant}\\ \text{variance}\end{pmatrix} + \begin{pmatrix}\text{Between}-\text{participant}\\ \text{variance}\end{pmatrix}}}$$

- These quantities can be estimated from single-case data using a hierarchical linear model.

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2.1 Meta-analysis of Study-Level Effect Sizes

D-CES can be estimated using two-level hierarchical modeling:

D-CES using basic two-level model

Level 1: $Y_{ij} = \beta_{0j} + \beta_{1j} Trt_{ij} + e_{ij}$

β_{1j} indicates the individual-specific intervention effect.

Level 2:

$$\beta_{0j} = \theta_0 + u_j$$
$$\beta_{1j} = \theta_1$$

θ_1 indicates the unstandardized intervention effect across the *J* cases.

D-CES

$$\delta = \frac{\theta_1}{\sqrt{\sigma^2 + \tau^2}}$$

Numerator: Unstandardized intervention effect

Denominator: The standard deviation of the outcome, including both within- and between-case variation

- *j* indicates the case (*j* = 1 to *J*), and case *j* is measured for a total of n_m measurement occasions (*i* = 1, … *I*).
- Y_{ij} indicates the outcome for case *j* at measurement occasion *i*.
- Trt_{ij} is a dummy variable indicating whether Y_{ij} is obtained during the baseline or the intervention phase.
- $e_{ij} \sim N(0, \sigma^2)$ and the errors for case *j* follow an *AR(1)* process; $u_j \sim N(0, \tau^2)$.

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2.1 Meta-analysis of Study-Level Effect Sizes

D-CES needs to be corrected for small-sample bias:

$$J(v) = 1 - \frac{3}{4v - 1}$$

where v is an estimated degree of freedom (this will be somewhere between the number of cases and the total number of time points; computation of v is different for AB^k and MB).

Bias-corrected D-CES:

$$g = J(v) \times \hat{\delta}$$

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2.1 Meta-analysis of Study-Level Effect Sizes

Meta-analysis methods:

- Random-effects meta-analysis model:

$$g_j = \beta + u_j + e_{ij}$$

where

$$u_j \sim N(0, \tau^2), \qquad e_{ij} \sim N\left(0, SE_{ij}^2\right)$$

Between-study variance (τ^2) describes heterogeneity of effects across studies.

- Can be estimated using the metafor package in R.

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2.1 Meta-analysis of Study-Level Effect Sizes

Limitations:

- Only one available metric (D-CES), based on models with normally distributed errors.
- Requires designs with **3+ participants** in order to estimate between-person variation in outcome (for scale).
- Limited available designs:
  - Across-participant multiple baseline/multiple probe
  - Replicated treatment reversals (ABAB)
  - Multiple baselines across behaviors, replicated across participants (Chen et al., 2022)
  - Clustered multiple baseline designs (Chen et al., 2022)
  - Multivariate across-participant multiple baseline designs (Chen et al., 2022)

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2.1 Meta-analysis of Study-Level Effect Sizes – Illustration

Data organization:

	A	B	C	D	E
1	Study_Name	Study	g	SE	
2	3.Bethune & Wood, 2013	3	2.88282	0.494453	
3	7.Boulineau et al., 2004	7	3.147	0.41	
4	9.Carnahan & Williamson, 2013	9	2.855	0.667	
5	24.Grunke et al., 2013	24	5.722	0.516	
6	29.Idoll & Croll, 1987	29	1.426	0.35	
7	38.Onachukwu et al., 2007	38	8.292	1.54	
8	43.Stetter & Hughes, 2011	43	-0.063	0.127	
9	45.Stringfield et al., 2011	45	3.807	0.347	
10	49.Vallecorsa & deBettebcourt, 1997	49	1.816	0.502	
11	53.Zakas et al., 2013	53	2.762	0.644	
12					
13					

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2.1 Meta-analysis of Study-Level Effect Sizes – Illustration

Results of random effects meta-analysis:

Average BC-SMD: $\hat{g} = 3.14$, SE = 0.69, 95% CI [1.79, 4.49], Z = 4.55

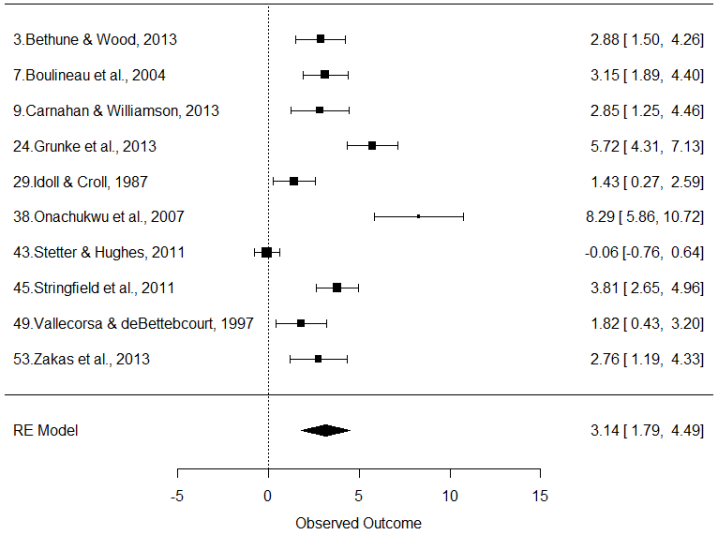
Between-study variance: $\tau^2 = 4.22$, SE = 2.24

Test for heterogeneity: Q(9) = 102.17, $p < .0001$

R output

```
Random-Effects Model (k = 10; tau^2 estimator: REML)

  logLik  deviance       AIC       BIC      AICc
-20.0734   40.1468   44.1468   44.5412   46.1468

tau^2 (estimated amount of total heterogeneity): 4.2254 (SE = 2.2410)
tau (square root of estimated tau^2 value):      2.0556
I^2 (total heterogeneity / total variability):   91.09%
H^2 (total variability / sampling variability):  11.22

Test for Heterogeneity:
Q(df = 9) = 102.1734, p-val < .0001

Model Results:

estimate      se    zval    pval   ci.lb   ci.ub
  3.1422  0.6900  4.5538  <.0001  1.7898  4.4946  ***

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Forest plot



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The What Works Clearinghouse (WWC) hosted a webinar (presenter Mariola Moeyaert) on October 13, 2020, to discuss the use, application, and estimation of the between-case standardized mean difference (BC-SMD) for single-case design studies (SCDs). For more details, see https://youtu.be/uXTbL8QkNvY.

2.2 Meta-analysis of Case-Level Effect Sizes

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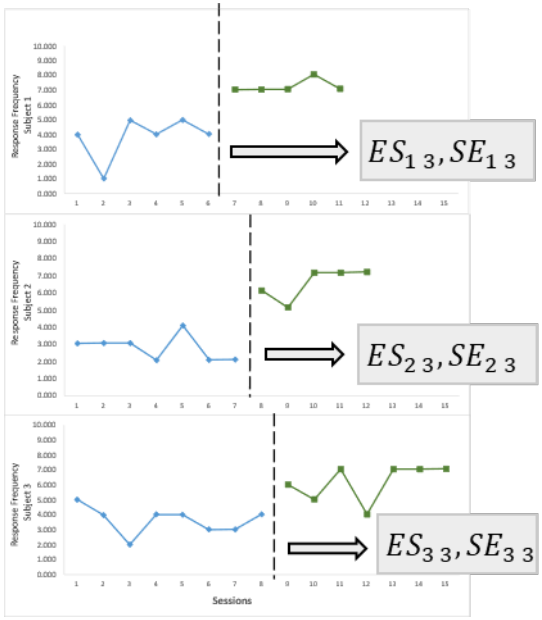
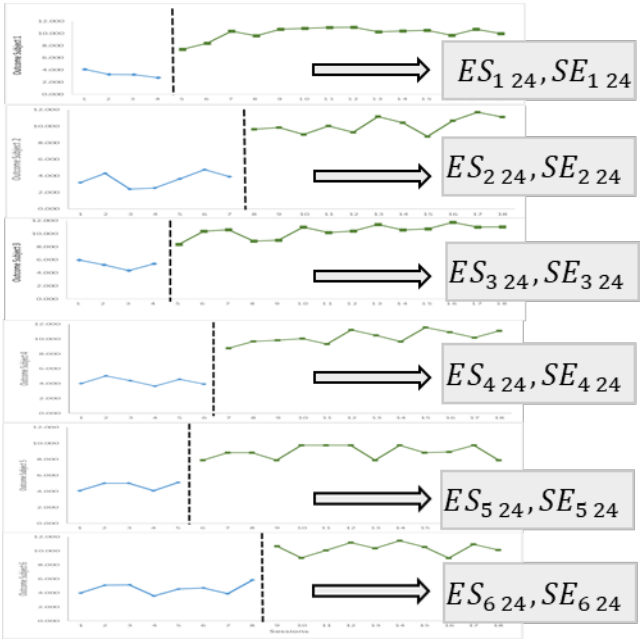
2.2 Meta-analysis of Case-Level Effect Sizes

Effect size: Single-number summary of intervention effect ***for each case.***

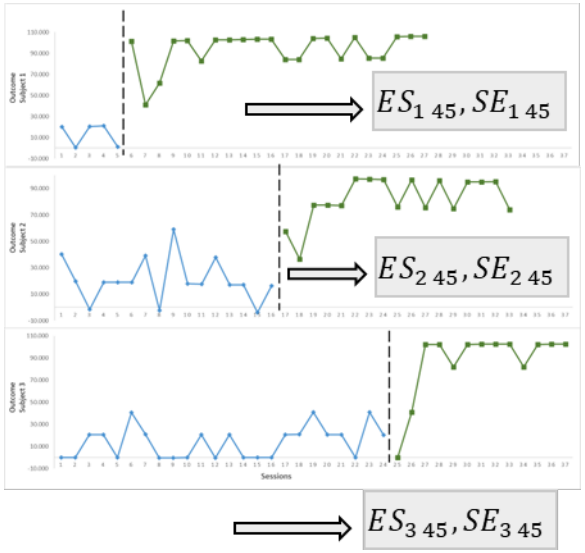
3. Bethune & Wood, 2013



24. Grünke et al., 2013



45. Stringfield et al., 2011



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2.2 Meta-analysis of Case-Level Effect Sizes

Goal: Compare results across participants and SCD studies ***that use various outcome measures***.

- Examine heterogeneity of effects within and between studies.
- Examine case-level predictors of effects.
- Available **ES metrics**: many, some appropriate for non-normal outcome distributions.
  - SingleCaseES web app and R package (https://www.jepusto.com/software/SingleCaseES/)
- Modeling **assumptions**:
  - Are case specific.
  - Most available ES assume no time trends.
  - Standard errors assume no autocorrelation.

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2.2 Meta-analysis of Case-Level Effect Sizes

Meta-analysis methods:

- Case-level ES estimates have a ***hierarchical structure***.
  - Effect size estimates for each case (sometimes multiple ES per case)
  - Multiple cases nested in each study

- This suggests that ES from the same study will be ***dependent***.
  - Can't use basic meta-analysis model (assumes all ES are independent)

- Two possible approaches to summarizing ES (Chen & Pustejovsky, in press):
  - Simple average/basic linear regression + Cluster-robust variance estimation
  - Multilevel meta-analysis/meta-regression + Cluster-robust variance estimation

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2.2 Meta-analysis of Case-Level Effect Sizes

Meta-analysis methods:

- Simple average: $ES_{ij} = \beta + \epsilon_{ij}$
  - Cluster-robust variance estimation accounts for dependent, heteroskedastic errors.
  - Provides results describing average effects and possible moderators, but not heterogeneity of effects.
  - Estimated using clubSandwich package in R.

- Multilevel meta-analysis: $ES_{ij} = \beta + u_j + v_{ij} + e_{ij}$

where $u_j \sim N(0, \tau^2),\ v_{ij} \sim N(0, \omega^2),\ e_{ij} \sim N\left(0, SE_{ij}^2\right)$

  - Variance components describe heterogeneity of effects across studies (τ^2) and across cases nested within studies (ω^2).
  - Cluster-robust variance estimation to account for dependency, possible misestimation of SE_{ij} (due to autocorrelation, small sample size, etc.).
  - Estimated using metafor package in R.

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2.2 Meta-analysis of Case-Level Effect Sizes

Available effect size metrics:

- Non-overlap indices (e.g., PND, NAP, Tau, PEN, PAND)
- Mean-based indices (e.g., log response ratio, within-case SMD)
- Regression-based effect sizes

2. Approaches to SCD Meta-analysis

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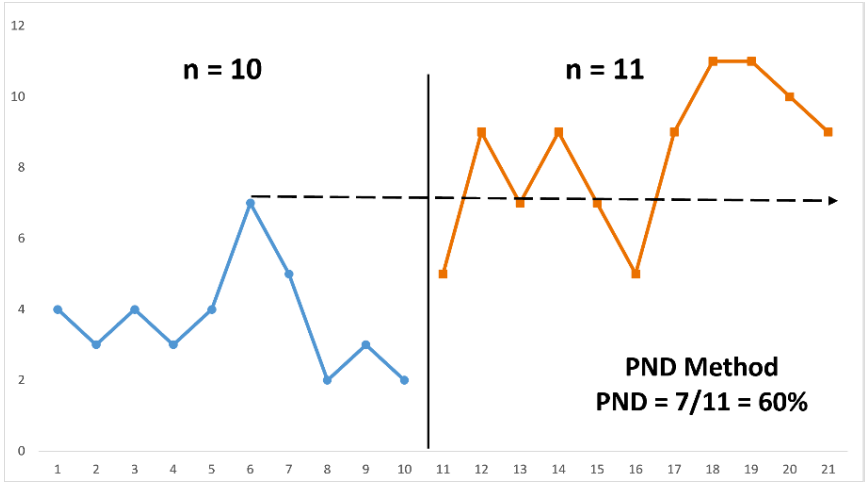
2.2 Meta-analysis of Case-Level Effect Sizes

Available effect size metrics:

- ***Non-overlap indices***
  (e.g., PND, NAP, Tau, PEN, PAND)



- Calculation of non-overlap between baseline and successive intervention phases → derivation of a percentage score.
- Easy to interpret: The higher the percentage, the more effective the treatment.
- Nonparametric (scores usually not independent and normally distributed).
- Developed without reference to parametric distributional modeling assumptions → lack known sampling variances (Shadish et al., 2008).
- Not reflecting magnitude of the effect.

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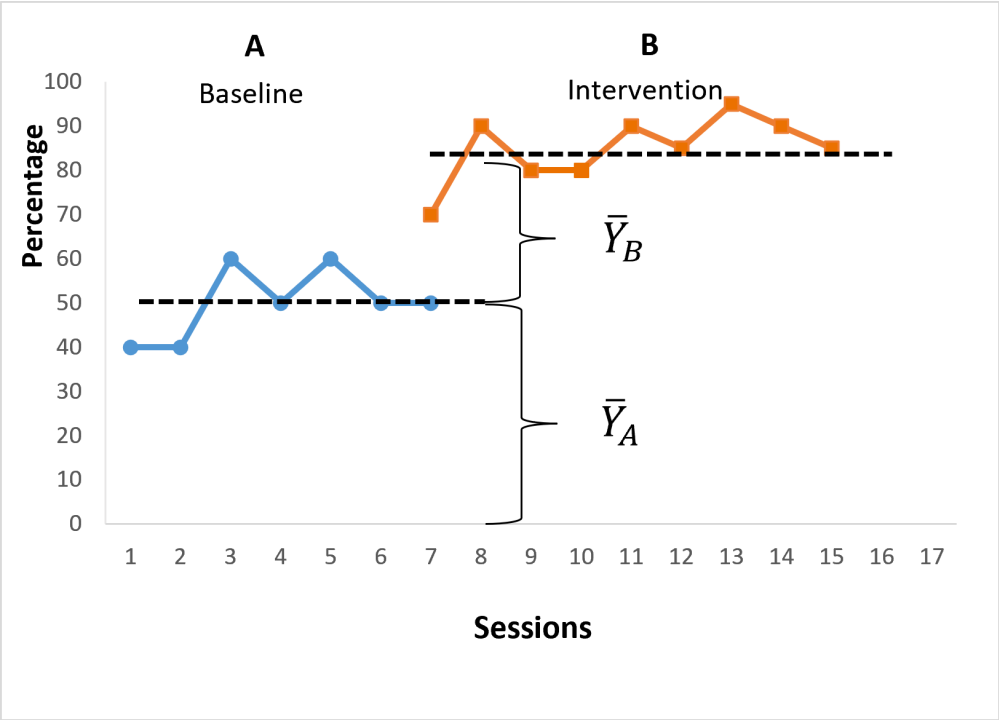
2.2 Meta-analysis of Case-Level Effect Sizes

Available effect size metrics:

- **Mean-based indices –** log response ratio

$$R = \ln\left(\frac{\bar{Y}_B}{\bar{Y}_A}\right)$$



– The natural logarithm is used to make distribution easier to work with.

– If the means are equal, then $R = 0$.

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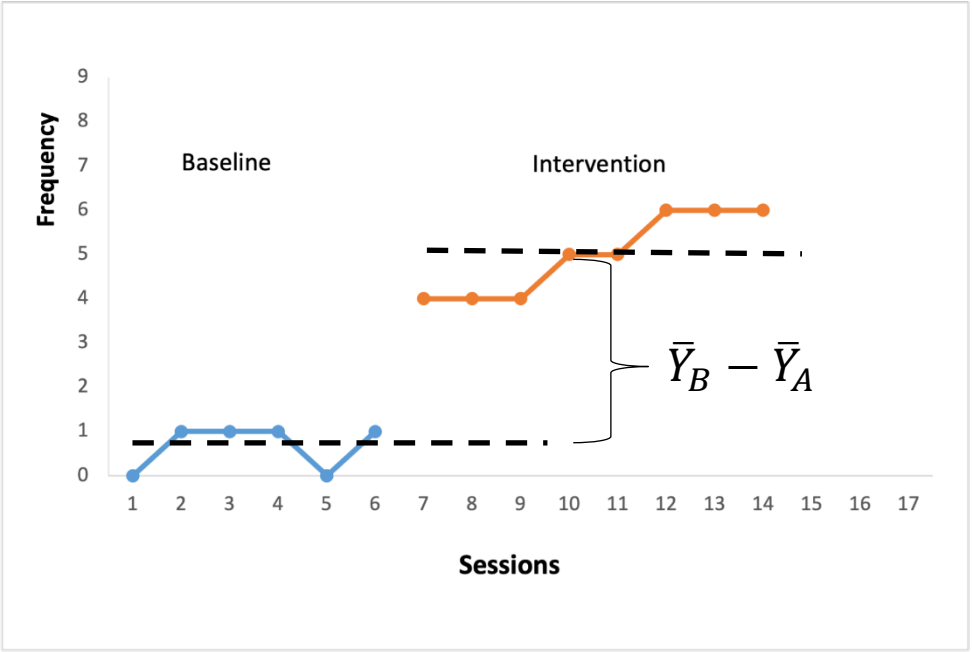
2.2 Meta-analysis of Case-Level Effect Sizes

Available effect size metrics:

- **Mean-based indices:** Within-case standardized mean difference (SMD_W)

$$SMD_W = \left(1 - \frac{3}{4m - 5}\right) \times \frac{\bar{Y}_B - \bar{Y}_A}{s_A}$$



– The SMD_W is standardized by the baseline standard deviation (s_A) and corrected for small *n* bias.

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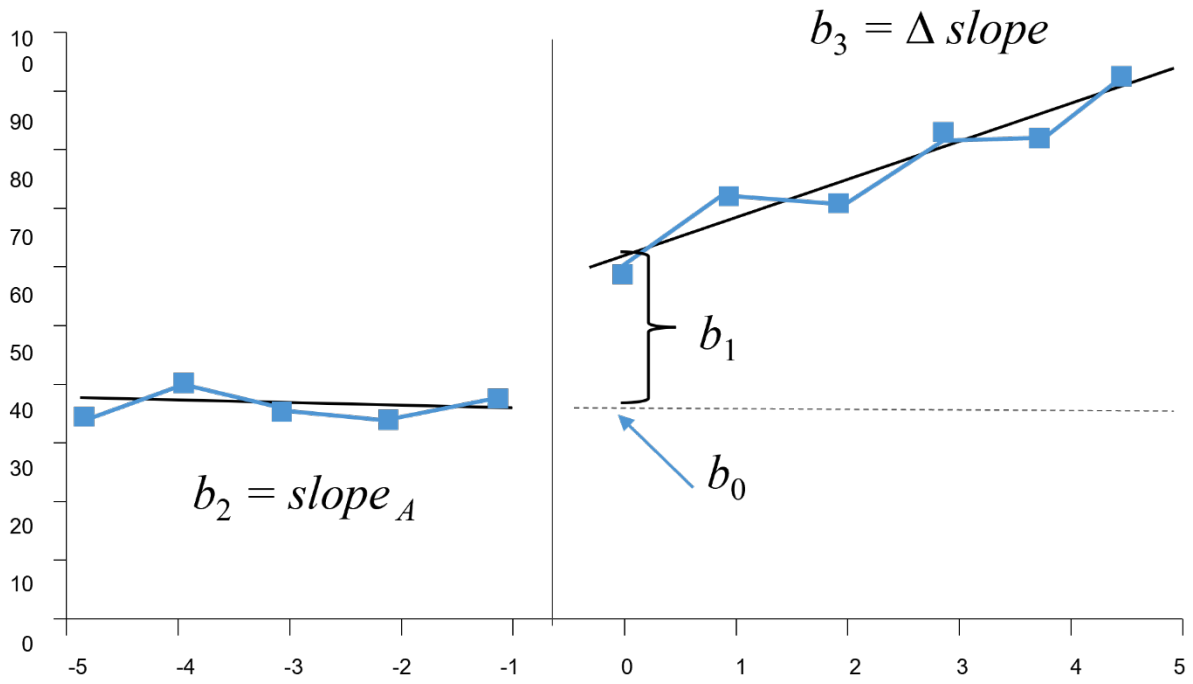
2.2 Meta-analysis of Case-Level Effect Sizes

Available effect size metrics:

- Regression-based effect sizes

$$y = \beta_0 + (\beta_1 \times Trt) + (\beta_2 \times \text{Time}) + (\beta_3 \times Trt \times \text{Time}) + e$$



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2.2 Meta-analysis of Case-Level Effect Sizes

Meta-analysis method depends on the effect size metric; see Chen & Pustejovsky (in press).

Metric	Strategy	Non-normal outcomes	Auto-correlation	Time trends
Log response ratio	Multilevel meta-analysis			
Within-case SMD	Simple average			
Non-overlap of All Pairs	Simple average			
Tau(AB)	Simple average			

2. Approaches to SCD Meta-analysis

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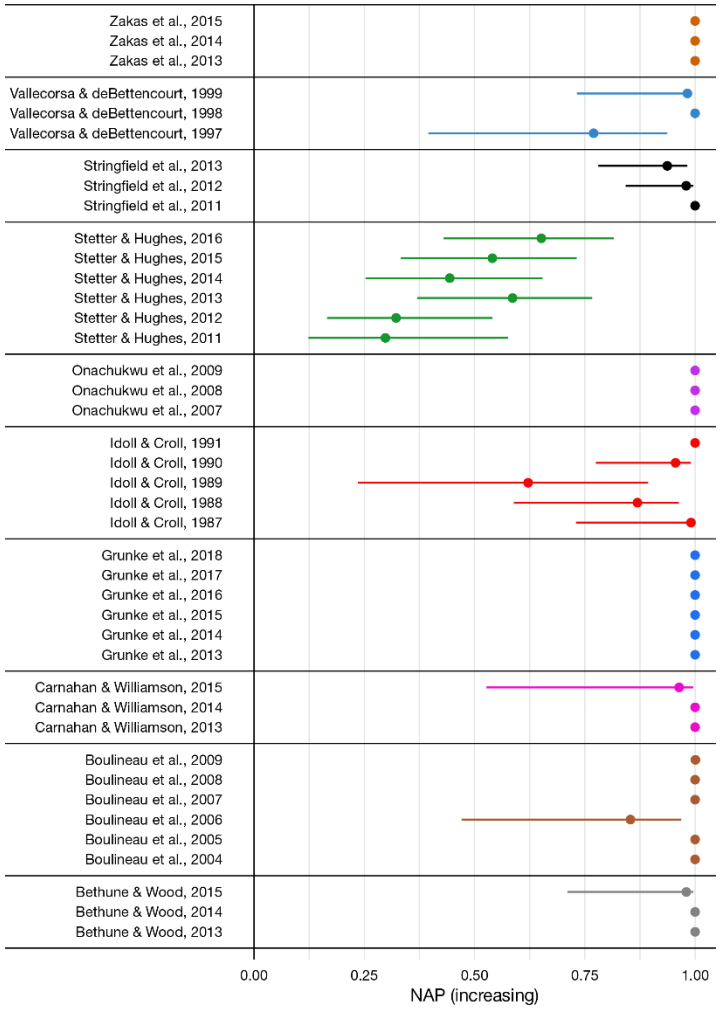
2.2 Meta-analysis of Case-Level Effect Sizes – Illustration

Data organization:

	Study	Case	ES	Est	SE	CI_lower	CI_upper
1	3	1	NAP	1.0000000	0.028625940	1.0000000	1.0000000
2	3	2	NAP	1.0000000	0.024222607	1.0000000	1.0000000
3	3	3	NAP	0.9821429	0.017857143	0.7132141	0.9990405
4	7	1	NAP	1.0000000	0.036877550	1.0000000	1.0000000
5	7	2	NAP	1.0000000	0.036877550	1.0000000	1.0000000
6	7	3	NAP	0.8541667	0.148195160	0.4718294	0.9713472
7	7	4	NAP	1.0000000	0.036877550	1.0000000	1.0000000
8	7	5	NAP	1.0000000	0.036877550	1.0000000	1.0000000
9	7	6	NAP	1.0000000	0.036877550	1.0000000	1.0000000
10	9	1	NAP	1.0000000	0.063464776	1.0000000	1.0000000
11	9	2	NAP	1.0000000	0.063464776	1.0000000	1.0000000
12	9	3	NAP	0.9666667	0.056519417	0.5292570	0.9984274
13	24	1	NAP	1.0000000	0.015062989	1.0000000	1.0000000
14	24	2	NAP	1.0000000	0.010369298	1.0000000	1.0000000
15	24	3	NAP	1.0000000	0.015062989	1.0000000	1.0000000
16	24	4	NAP	1.0000000	0.011197580	1.0000000	1.0000000
17	24	5	NAP	1.0000000	0.012610459	1.0000000	1.0000000
18	24	6	NAP	1.0000000	0.009929064	1.0000000	1.0000000
19	29	1	NAP	0.9932432	0.006756757	0.7326183	0.9998411
20	29	2	NAP	0.8717949	0.088823118	0.5909598	0.9658910
21	29	3	NAP	0.6250000	0.246503324	0.2388335	0.8948526
22	29	4	NAP	0.9583333	0.028815170	0.7778645	0.9925410
23	29	5	NAP	1.0000000	0.028625940	1.0000000	1.0000000



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2.2 Meta-analysis of Case-Level Effect Sizes – Illustration

Output:

$\text{NAP} = .94, \text{SE} = 0.049, t(8.99) = 19.36, p < .001$

$\omega^2 = 0.0238$ and $\tau^2 = 0.00$

- The NAP is very large.
- There is little between-participant and between-study variability in intervention effectiveness.

```
Multivariate Meta-Analysis Model (k = 41; method: REML)

   logLik   Deviance        AIC        BIC       AICc
  62.8102  -125.6204  -119.6204  -114.5538  -118.9538

Variance Components:

            estim    sqrt  nlvls  fixed      factor
sigma^2.1  0.0238  0.1542     10     no       Study
sigma^2.2  0.0000  0.0000     41     no  Study/Case

Test for Heterogeneity:
Q(df = 40) = 153.7559, p-val < .0001

Number of estimates:   41
Number of clusters:    10
Estimates per cluster: 3-6 (mean: 4.10, median: 3)

Model Results:

estimate      se'     tval'    df'    pval'   ci.lb'   ci.ub'
  0.9463  0.0489   19.3566   8.99   <.0001   0.8357   1.0569   ***

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

1) results based on cluster-robust inference (var-cov estimator: CR2,
   approx t-test and confidence interval, df: Satterthwaite approx)
```

2.3 Raw Data Meta-analysis

2. Approaches to SCD Meta-analysis

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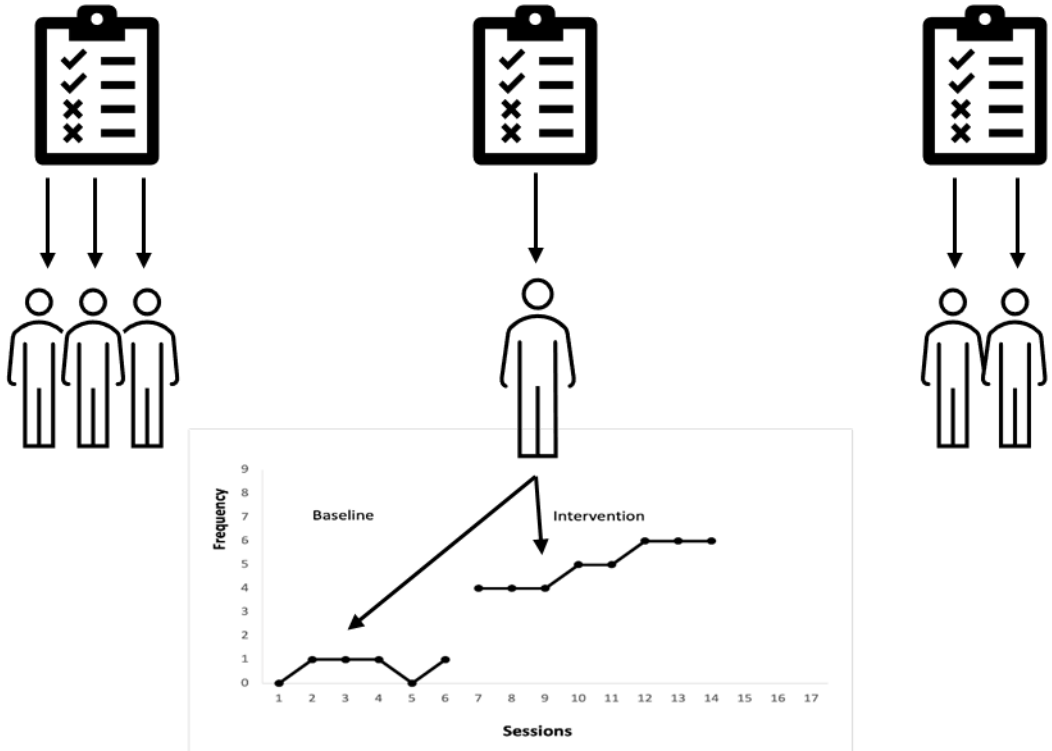
2.3 Raw Data Meta-analysis

- Raw SCD data meta-analysis is also called raw individual patient/participant data (IPD) meta-analysis (Declercq et al., 2020; Moeyaert & Fingerhut, 2022).
- Raw data from multiple participants and studies are synthesized.
- Three-level structure:

Level 3:
Studies

Level 2:
Cases

Level 1:
Measurements



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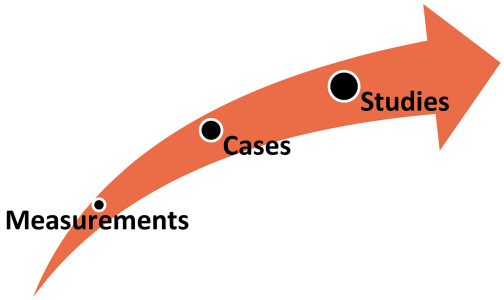
2.3 Raw Data Meta-analysis



Goals:

- Modeling the three-level hierarchical structure
- Estimating the overall intervention effect across cases and across studies in addition to participant-specific and study-specific treatment effects
- Estimating between-participant and between-study variance
- Investigating moderators at both the case and study levels to explain intervention heterogeneity

Source: Moeyaert & Yang (2021)

2. Approaches to SCD Meta-analysis

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2.3 Raw Data Meta-analysis

Meta-analysis method: IPD meta-analysis

Level 1: Variation of scores within participants

$$y_{ijk} = \boldsymbol{\beta_{0jk}} + \boldsymbol{\beta_{1jk}} \times Trt_{ijk} + e_{ijk}$$

Level 2: Variation between participants from the same study

$$\boldsymbol{\beta_{0jk}} = \boldsymbol{\theta_{00k}} + u_{0jk}$$

$$\boldsymbol{\beta_{1jk}} = \boldsymbol{\theta_{10k}} + u_{1jk}$$

$$\begin{pmatrix} u_{0jk} \\ u_{1jk} \end{pmatrix} \sim MVN\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_{u_0} & \\ \sigma_{u_{01}} & \sigma^2_{u_1} \end{pmatrix}\right]$$

Level 3: Variation between studies

$$\boldsymbol{\theta_{00k}} = \boldsymbol{\gamma_{000}} + v_{00k}$$

$$\boldsymbol{\theta_{10k}} = \boldsymbol{\gamma_{100}} + v_{10k}$$

$$\begin{pmatrix} v_{0jk} \\ v_{1jk} \end{pmatrix} \sim MVN\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_{v_0} & \\ \sigma_{v_{01}} & \sigma^2_{v_1} \end{pmatrix}\right]$$

2. Approaches to SCD Meta-analysis

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2.3 Raw Data Meta-analysis

Meta-analysis method: IPD meta-analysis

Combined model:

Overall intervention effect

$$y_{ijk} = \boldsymbol{\gamma_{000}} + u_{0jk} + v_{00k} + (\boldsymbol{\gamma_{100}} + u_{1jk} + v_{10k}) \times Trt_{ijk} + e_{ijk}$$

with $e_{ijk} \sim N\,(0, \sigma_e^2)$ and

Between-case variance in intervention effect

$$\begin{pmatrix} u_{0jk} \\ u_{1jk} \end{pmatrix} \sim MVN\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{u_0}^2 & \\ \sigma_{u_{01}} & \sigma_{u_1}^2 \end{pmatrix}\right]$$

Between-study variance in intervention effect

$$\begin{pmatrix} v_{0jk} \\ v_{1jk} \end{pmatrix} \sim MVN\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{v_0}^2 & \\ \sigma_{v_{01}} & \sigma_{v_1}^2 \end{pmatrix}\right]$$

Meta-analysts are interested in the **estimate of $\boldsymbol{\gamma_{100}}$**, which expresses the overall intervention effect across participants and studies; and in the variance component $\boldsymbol{\sigma_{u_1}^2}$, which expresses the extent to which the intervention effect varies across participants within the study; and in the variance component $\boldsymbol{\sigma_{v_1}^2}$, which expresses the extent to which the intervention varies across studies.

2. Approaches to SCD Meta-analysis

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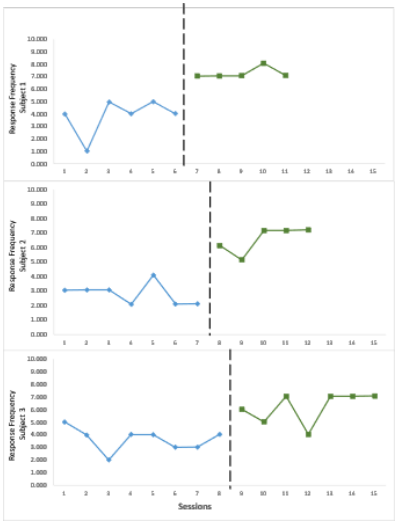
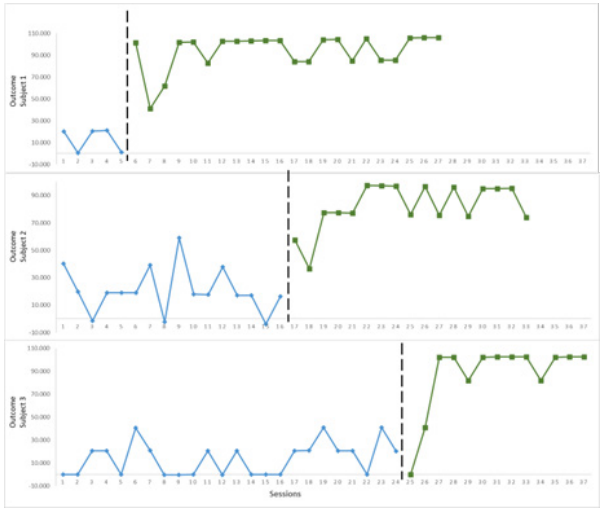
2.3 Raw Data Meta-analysis

- Example of two studies included in empirical illustration:

Outcome scale: 0–10



Outcome scale: 0–100



- Therefore, ***standardization is needed*** (Moeyaert et al., 2013; Van den Noortgate & Onghena, 2008).

- The outcome data ($y'_{ijk}s$) are standardized by dividing them by the estimated residual within-subject standard deviation of participant j from study k, $\hat{\sigma}_{ejk}$ (Van den Noortgate & Onghena, 2008):

  $y'_{ijk} = \frac{y_{ijk}}{\hat{\sigma}_{ejk}}$, $\hat{\sigma}_{ejk}$ is obtained by running the following OLS per participant: $y_{ijk} = \beta_{0jk} + \beta_{1jk} \times Trt_{ijk} + e_{ijk}$

2. Approaches to SCD Meta-analysis

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2.3 Raw Data Meta-analysis

The major advantage of using IPD meta-analysis is its flexibility.

Variety of modeling options:

- Heterogeneity of variances
- Autocorrelation
- Count outcomes
- Nonlinear time trend
- Multiple moderators

The major limitation is modeling assumptions:

- Mis-specification of data trends
- Normality of level 1, level 2, and level 3 residuals

2. Approaches to SCD Meta-analysis

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2.3 Raw Data Meta-analysis – Illustration

Data organization:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
	Study_Name	Study	Case	Outcomes	Session	Trt	Interventionist	Digital	Disability_1	Disability_2	Disability_3	Quality	Treatment_integrity	Dosage	Social_Validity	Age	Grade	Gender	IQ	school	Yea	Mean Square	RMSE	yS
	3.Bethune & Wood, 2013	3	1	3.997	1	0	1	0	4	0	0	2	1	5	1	8	NA	1	94	1	201	1.28409	1.1332	3.527
	3.Bethune & Wood, 2013	3	1	1.011	2	0	1	0	4	0	0	2	1	5	1	8	NA	1	94	1	201	1.28409	1.1332	0.892
	3.Bethune & Wood, 2013	3	1	4.972	3	0	1	0	4	0	0	2	1	5	1	8	NA	1	94	1	201	1.28409	1.1332	4.388
	3.Bethune & Wood, 2013	3	1	4.022	4	0	1	0	4	0	0	2	1	5	1	8	NA	1	94	1	201	1.28409	1.1332	3.549
	3.Bethune & Wood, 2013	3	1	5.001	5	0	1	0	4	0	0	2	1	5	1	8	NA	1	94	1	201	1.28409	1.1332	4.413
	3.Bethune & Wood, 2013	3	1	4.032	6	0	1	0	4	0	0	2	1	5	1	8	NA	1	94	1	201	1.28409	1.1332	3.558
	3.Bethune & Wood, 2013	3	1	7.047	7	1	1	0	4	0	0	2	1	5	1	8	NA	1	94	1	201	1.28409	1.1332	6.219
	3.Bethune & Wood, 2013	3	1	7.061	8	1	1	0	4	0	0	2	1	5	1	8	NA	1	94	1	201	1.28409	1.1332	6.231
0	3.Bethune & Wood, 2013	3	1	7.075	9	1	1	0	4	0	0	2	1	5	1	8	NA	1	94	1	201	1.28409	1.1332	6.244
1	3.Bethune & Wood, 2013	3	1	8.089	10	1	1	0	4	0	0	2	1	5	1	8	NA	1	94	1	201	1.28409	1.1332	7.138
2	3.Bethune & Wood, 2013	3	1	7.104	11	1	1	0	4	0	0	2	1	5	1	8	NA	1	94	1	201	1.28409	1.1332	6.269
3	3.Bethune & Wood, 2013	3	2	3.064	1	0	1	0	4	0	0	2	1	5	1	10	NA	1	67	1	201	0.67664	0.8226	3.725
4	3.Bethune & Wood, 2013	3	2	3.082	2	0	1	0	4	0	0	2	1	5	1	10	NA	1	67	1	201	0.67664	0.8226	3.747
5	3.Bethune & Wood, 2013	3	2	3.083	3	0	1	0	4	0	0	2	1	5	1	10	NA	1	67	1	201	0.67664	0.8226	3.748
5	3.Bethune & Wood, 2013	3	2	2.092	4	0	1	0	4	0	0	2	1	5	1	10	NA	1	67	1	201	0.67664	0.8226	2.543
7	3.Bethune & Wood, 2013	3	2	4.111	5	0	1	0	4	0	0	2	1	5	1	10	NA	1	67	1	201	0.67664	0.8226	4.998
8	3.Bethune & Wood, 2013	3	2	2.111	6	0	1	0	4	0	0	2	1	5	1	10	NA	1	67	1	201	0.67664	0.8226	2.566
9	3.Bethune & Wood, 2013	3	2	2.129	7	0	1	0	4	0	0	2	1	5	1	10	NA	1	67	1	201	0.67664	0.8226	2.588
0	3.Bethune & Wood, 2013	3	2	6.149	8	1	1	0	4	0	0	2	1	5	1	10	NA	1	67	1	201	0.67664	0.8226	7.475
1	3.Bethune & Wood, 2013	3	2	5.158	9	1	1	0	4	0	0	2	1	5	1	10	NA	1	67	1	201	0.67664	0.8226	6.271
2	3.Bethune & Wood, 2013	3	2	7.196	10	1	1	0	4	0	0	2	1	5	1	10	NA	1	67	1	201	0.67664	0.8226	8.748
3	3.Bethune & Wood, 2013	3	2	7.197	11	1	1	0	4	0	0	2	1	5	1	10	NA	1	67	1	201	0.67664	0.8226	8.749
4	3.Bethune & Wood, 2013	3	2	7.232	12	1	1	0	4	0	0	2	1	5	1	10	NA	1	67	1	201	0.67664	0.8226	8.792
5	3.Bethune & Wood, 2013	3	3	5.018	1	0	1	0	4	0	0	2	1	7	1	10	NA	1	90	1	201	1.14492	1.07	4.69
5	3.Bethune & Wood, 2013	3	3	3.99	2	0	1	0	4	0	0	2	1	7	1	10	NA	1	90	1	201	1.14492	1.07	3.729
7	3.Bethune & Wood, 2013	3	3	2.015	3	0	1	0	4	0	0	2	1	7	1	10	NA	1	90	1	201	1.14492	1.07	1.883
8	3.Bethune & Wood, 2013	3	3	4.022	4	0	1	0	4	0	0	2	1	7	1	10	NA	1	90	1	201	1.14492	1.07	3.759
9	3.Bethune & Wood, 2013	3	3	4.011	5	0	1	0	4	0	0	2	1	7	1	10	NA	1	90	1	201	1.14492	1.07	3.749
0	3.Bethune & Wood, 2013	3	3	3.018	6	0	1	0	4	0	0	2	1	7	1	10	NA	1	90	1	201	1.14492	1.07	2.821
1	3.Bethune & Wood, 2013	3	3	3.026	7	0	1	0	4	0	0	2	1	7	1	10	NA	1	90	1	201	1.14492	1.07	2.828
2	3.Bethune & Wood, 2013	3	3	4.033	8	0	1	0	4	0	0	2	1	7	1	10	NA	1	90	1	201	1.14492	1.07	3.769
3	3.Bethune & Wood, 2013	3	3	6.023	9	1	1	0	4	0	0	2	1	7	1	10	NA	1	90	1	201	1.14492	1.07	5.629
4	3.Bethune & Wood, 2013	3	3	5.03	10	1	1	0	4	0	0	2	1	7	1	10	NA	1	90	1	201	1.14492	1.07	4.701
5	3.Bethune & Wood, 2013	3	3	7.055	11	1	1	0	4	0	0	2	1	7	1	10	NA	1	90	1	201	1.14492	1.07	6.593
5	3.Bethune & Wood, 2013	3	3	4.045	12	1	1	0	4	0	0	2	1	7	1	10	NA	1	90	1	201	1.14492	1.07	3.781

2. Approaches to SCD Meta-analysis

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2.3 Raw Data Meta-analysis – Illustration

Results of the raw data meta-analysis (i.e., three-level hierarchical linear modeling):

$\gamma_{100} = 8.26, SE = 4.14, t(719) = 1.99, p = .0461$

$\sigma^2_{u_1} = 25.11^2$ **and** $\sigma^2_{v_1} = 4.11^2$

- There is a substantial amount of between-participant and between-study variability in intervention effectiveness.
- Most variability is between participants.

```
Linear mixed-effects model fit by REML
  Data: IPD_standardized
       AIC      BIC    logLik
  2684.048 2725.736 -1333.024

Random effects:
 Formula: ~1 + Trt | Study
 Structure: General positive-definite, Log-Cholesky parametrization
            StdDev    Corr
(Intercept) 0.6719678 (Intr)
Trt         4.1065682 -0.523

 Formula: ~1 + Trt | Case %in% Study
 Structure: General positive-definite, Log-Cholesky parametrization
            StdDev    Corr
(Intercept) 13.744339 (Intr)
Trt         25.107137 0.989
Residual     1.000923

Fixed effects:  yS ~ 1 + Trt
                Value Std.Error  DF  t-value p-value
(Intercept) 5.737765  2.158667 719 2.658013  0.0080
Trt         8.263519  4.135315 719 1.998280  0.0461
 Correlation:
    (Intr)
Trt 0.916

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-3.96522921 -0.61186911  0.04172565  0.68551833  2.51193279

Number of Observations: 761
Number of Groups:
          Study Case %in% Study
             10              41
```

3. Concluding Thoughts

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- Currently, the three different SCD meta-analytic approaches are not well understood/disseminated.
- The appropriate meta-analytic approach depends on the research question, level of analysis, available effect metrics, modeling assumptions.
- Both applied SCD researchers and methodologists need to work on *clarifying* research questions, level of analysis, available effect metrics, and modeling assumptions. Through their efforts, the most appropriate approach can be rationalized.

References

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Additional Resources

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Evidence-Based Communication Assessment and Intervention, Vol. 14, 1–2 & Vol. 17, 1–2

#1 – Advances in Statistical Analysis and Meta-analysis of Single-Case Experimental Designs

#2 – New Developments in Meta-analysis of Single-Case Experimental Designs

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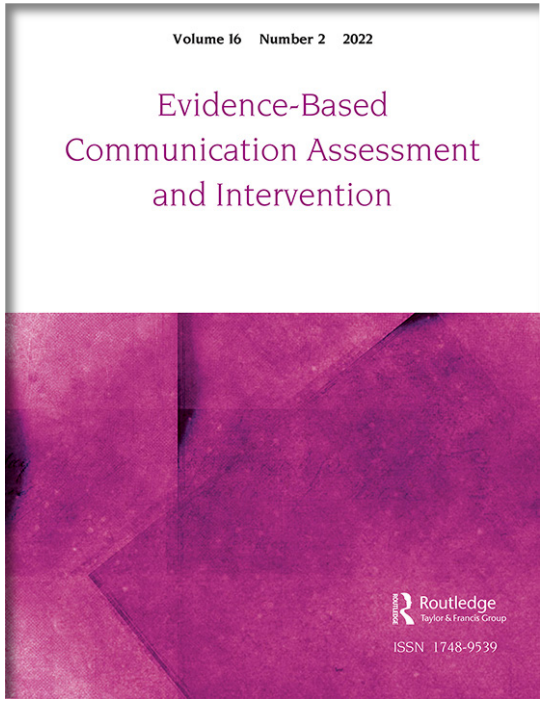

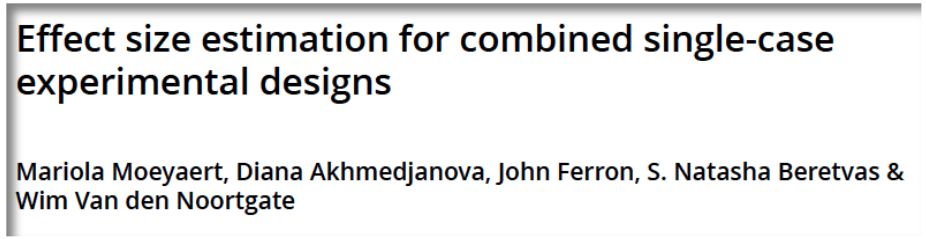

Follow-up Webinar

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Fall 2023:

Meta-analysis of Single-Case Experimental Designs: Applications of Advanced Methods for Synthesizing Group and Single-Case Data

Speakers:

Mariola Moeyaert, Ralf Schlosser, & Oliver Wendt

Campbell Collaboration

Campbell Webinar Series 2023

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Meta-analysis of Single-Case Experimental Designs:
An Overview of Current Synthesis Methods

Questions?

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