Meta-analysis of Single-Case Experimental Designs

An Overview of Current Synthesis Methods

Campbell Webinar Series 2023
February 7, 2023

A webcast co-sponsored by AIR’s Center on KTDRR
Acknowledgments

1. Through support from the Institute of Education Sciences (IES), U.S. Department of Education, I was able to study single-case experimental design (SCD) meta-analysis approaches over the last decade.

The opinions expressed herein are those of the authors and do not represent the views of IES or the U.S. Department of Education.

• R305D110024, Multilevel synthesis of single-case experimental data: Further developments and empirical validation

• R305D150007, Multilevel modeling of single-subject experimental data: Handling data and design complexities

• R305D190022, Assessing generalizability and variability of single-case design effect sizes using multilevel modeling, including moderators

2. Dr. James Pustejovsky, whose valuable collaboration on slide content created for the IES SCD research institute, informed the slides developed for this presentation.

Dr. Mariola Moeyaert
1. Introduction

2. Approaches to Meta-analysis of Single-Case Experimental Designs
   2.1 Meta-analysis of Study-Level Summary Effect Sizes
   2.2 Meta-analysis of Case-Level Effect Sizes
   2.3 Raw Data Meta-analysis

3. Concluding Thoughts

4. Questions and Answers
1. Introduction

Single-case experimental designs (SCDs) are

• designed experiments in which one unit is observed repeatedly during a certain period of time under different levels of at least one manipulated variable.

• experimental designs with the potential to demonstrate a causal effect.

The SCD logic has many variations, but all SCDs often involve repeated, systematic measurement of a dependent variable before, during, and/or after the active manipulation of an independent variable.
1. Introduction

SCD example:
Multiple baseline design –
Replication across subjects
1. Introduction

Exponential increase in popularity

Source: Web of Science, Keywords: TS = (single-case* OR single-subject* OR interrupted time series* of intra-subject* or n-of-1*); field = education.
Evidence-Based Education Policy

• We have entered an era in which scientific evidence will increasingly inform policy.

• Combining evidence from multiple SCD studies, using **meta-analytic techniques**, can provide a basis for generalization about effects of intervention.

• Using **meta-analysis**, the focus is on
  − *Summarizing magnitude* of intervention effects.
  − Investigating *intervention heterogeneity*.
  − Identifying *moderators* to explain intervention heterogeneity.
1. Introduction

Before we can get started:

- Raw SCD data from all participant graphs need to be retrieved.
- Data retrieval software programs can be used for this purpose (e.g., Moeyaert et al., 2016; WebPlotDigitizer).
2. Approaches to SCD Meta-analysis

Three broad approaches to meta-analysis:

2.1 Meta-analysis of study-level summary effect sizes
2.2 Meta-analysis of case-level effect sizes
2.3 Raw data meta-analysis (i.e., individual patient data meta-analysis)

These approaches differ in multiple respects:

• Level of analysis
• Available effect size metrics
• Modeling assumptions
2. Approaches to SCD Meta-analysis

Three broad approaches to meta-analysis:

SCD + group designs

Study types?

2.1 Study-level effect sizes (between-case SMD)

Varied/heterogeneous DVs

2.2 Case-level effect sizes

Common/equatable DVs

2.3 Raw data synthesis

SCDs exclusively

Outcome types?

Note. SMD is standardized mean difference; DV is dependent variable.
2. Approaches to SCD Meta-analysis

Data to illustrate the three broad approaches:

• Inclusion criteria:
  – Design needs to be an SCD.
  – Population is K–12 students with identified disability.
  – A graphic organizer intervention was implemented.
  – Outcome is reading comprehension.
  – The effect of the intervention on reading comprehension was reported graphically, with a clearly identifiable baseline phase and an intervention phase.

• A total of 23 SCD studies met the inclusion criteria.

• For this illustration, a random selection of 10 SCD studies was sampled.
2. Approaches to SCD Meta-analysis

Illustration of three broad approaches to meta-analysis:

- Raw meta-analytic dataset
2. Approaches to SCD Meta-analysis

2.1 Meta-analysis of Study-Level Effect Sizes
2. Approaches to SCD Meta-analysis

2.1 Meta-analysis of Study-Level Effect Sizes

**Effect size:** Between-case standardized mean difference (a.k.a. design-comparable effect size)

Single-number summary of average study intervention effect


45. Stringfield et al., 2011

Campbell Webinar

- $g_3, SE_3$
- $g_{24}, SE_{24} ...$
- $g_{45}, SE_{45}$
2. Approaches to SCD Meta-analysis

2.1 Meta-analysis of Study-Level Effect Sizes

**Goal:** Summarize study-level average ES (effect size) in a metric that is theoretically comparable to ES from a between-group design.

- Can then use conventional meta-analysis methods for synthesis.

Available **ES metric:** Between-case SMD (standardized mean difference), also called D-CES (design-comparable effect size).

- scdhlm web app and R package
  (https://www.jepusto.com/software/scdhlm/)
- Between-case SMD ES for SCDs: a primer and tutorial using the scdhlm web application

**Modeling assumptions:**
- Hierarchical model for data from each study
- Treatment initiation and follow-up times in hypothetical group design
2. Approaches to SCD Meta-analysis

2.1 Meta-analysis of Study-Level Effect Sizes

- What is the SMD from a between-groups experiment?

\[
\delta_{BC} = \frac{\text{Average outcome if everybody gets treatment} - \text{Average outcome if nobody gets treatment}}{\text{Outcome standard dev. (nobody gets treatment)}}
\]

\[
\delta_{BC} = \frac{\text{Average outcome if everybody gets treatment} - \text{Average outcome if nobody gets treatment}}{\sqrt{\text{Within-participant variance} + \text{Between-participant variance}}}
\]

- These quantities can be estimated from single-case data using a hierarchical linear model.
2. Approaches to SCD Meta-analysis

2.1 Meta-analysis of Study-Level Effect Sizes

D-CES can be estimated using two-level hierarchical modeling:

**D-CES using basic two-level model**

**Level 1:**
\[ Y_{ij} = \beta_{0j} + \beta_{1j} Trt_{ij} + e_{ij} \]

- \( \beta_{1j} \) indicates the individual-specific intervention effect.

**Level 2:**
\[ \begin{align*}
\beta_{0j} &= \theta_0 + u_j \\
\beta_{1j} &= \theta_1 
\end{align*} \]

- \( \theta_1 \) indicates the unstandardized intervention effect across the \( J \) cases.

**D-CES**
\[ \delta = \frac{\theta_1}{\sqrt{\sigma^2 + \tau^2}} \]

- Numerator: Unstandardized intervention effect
- Denominator: The standard deviation of the outcome, including both within- and between-case variation

- \( j \) indicates the case \( (j = 1 \) to \( J \), and case \( j \) is measured for a total of \( n_m \) measurement occasions \( (i = 1, \ldots, I) \).
- \( Y_{ij} \) indicates the outcome for case \( j \) at measurement occasion \( i \).
- \( Trt_{ij} \) is a dummy variable indicating whether \( Y_{ij} \) is obtained during the baseline or the intervention phase.
- \( e_{ij} \sim N(0, \sigma^2) \) and the errors for case \( j \) follow an AR(1) process; \( u_j \sim N(0, \tau^2) \).
2. Approaches to SCD Meta-analysis

2.1 Meta-analysis of Study-Level Effect Sizes

D-CES needs to be corrected for small-sample bias:

\[ J(v) = 1 - \frac{3}{4v - 1} \]

where \( v \) is an estimated degree of freedom (this will be somewhere between the number of cases and the total number of time points; computation of \( v \) is different for AB\(^k\) and MB).

Bias-corrected D-CES:

\[ g = J(v) \times \hat{\delta} \]
2. Approaches to SCD Meta-analysis

2.1 Meta-analysis of Study-Level Effect Sizes

Meta-analysis methods:

• Random-effects meta-analysis model:

\[ g_j = \beta + u_j + e_{ij} \]

where

\[ u_j \sim N(0, \tau^2), \quad e_{ij} \sim N(0, SE_{ij}^2) \]

Between-study variance (\( \tau^2 \)) describes heterogeneity of effects across studies.

• Can be estimated using the metafor package in R.
2. Approaches to SCD Meta-analysis

2.1 Meta-analysis of Study-Level Effect Sizes

Limitations:

• Only one available metric (D-CES), based on models with normally distributed errors.

• Requires designs with 3+ participants in order to estimate between-person variation in outcome (for scale).

• Limited available designs:
  – Across-participant multiple baseline/multiple probe
  – Replicated treatment reversals (ABAB)
  – Multiple baselines across behaviors, replicated across participants (Chen et al., 2022)
  – Clustered multiple baseline designs (Chen et al., 2022)
  – Multivariate across-participant multiple baseline designs (Chen et al., 2022)
2. Approaches to SCD Meta-analysis

2.1 Meta-analysis of Study-Level Effect Sizes – Illustration

Data organization:

<table>
<thead>
<tr>
<th>Study_Name</th>
<th>Study</th>
<th>g</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. Bethune &amp; Wood, 2013</td>
<td>3</td>
<td>2.88282</td>
<td>0.494453</td>
</tr>
<tr>
<td>7. Boulineau et al., 2004</td>
<td>7</td>
<td>3.147</td>
<td>0.41</td>
</tr>
<tr>
<td>9. Carnahan &amp; Williamson, 2013</td>
<td>9</td>
<td>2.855</td>
<td>0.667</td>
</tr>
<tr>
<td>24. Grunke et al., 2013</td>
<td>24</td>
<td>5.722</td>
<td>0.516</td>
</tr>
<tr>
<td>29. Idoll &amp; Croll, 1987</td>
<td>29</td>
<td>1.426</td>
<td>0.35</td>
</tr>
<tr>
<td>38. Onachukwu et al., 2007</td>
<td>38</td>
<td>8.292</td>
<td>1.54</td>
</tr>
<tr>
<td>43. Stetter &amp; Hughes, 2011</td>
<td>43</td>
<td>-0.063</td>
<td>0.127</td>
</tr>
<tr>
<td>45. Stringfield et al., 2011</td>
<td>45</td>
<td>3.807</td>
<td>0.347</td>
</tr>
<tr>
<td>49. Vallecorsa &amp; deBettebcourt, 1997</td>
<td>49</td>
<td>1.816</td>
<td>0.502</td>
</tr>
<tr>
<td>53. Zakas et al., 2013</td>
<td>53</td>
<td>2.762</td>
<td>0.644</td>
</tr>
</tbody>
</table>
2. Approaches to SCD Meta-analysis

2.1 Meta-analysis of Study-Level Effect Sizes – Illustration

Results of random effects meta-analysis:

Average BC-SMD: $\hat{g} = 3.14$, SE = 0.69, 95% CI [1.79, 4.49], Z = 4.55
Between-study variance: $\tau^2 = 4.22$, SE = 2.24
Test for heterogeneity: $Q(9) = 102.17$, $p < .0001$

R output

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>logLik</td>
<td>-20.0734</td>
</tr>
<tr>
<td>deviance</td>
<td>40.1468</td>
</tr>
<tr>
<td>AIC</td>
<td>44.1468</td>
</tr>
<tr>
<td>BIC</td>
<td>44.5412</td>
</tr>
<tr>
<td>AICc</td>
<td>46.1468</td>
</tr>
</tbody>
</table>

$tau^2$ (estimated amount of total heterogeneity): 4.2234 (SE = 2.2410)
$\tau$ (square root of estimated $tau^2$ value): 2.0556
$I^2$ (total heterogeneity / total variability): 91.09%
$H^2$ (total variability / sampling variability): 11.22

Test for Heterogeneity:
$Q(df = 9) = 102.1734$, p-val < .0001

Model Results:

<table>
<thead>
<tr>
<th>Estimate</th>
<th>se</th>
<th>zval</th>
<th>pval</th>
<th>ci.lb</th>
<th>ci.ub</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1422</td>
<td>0.6900</td>
<td>4.5538</td>
<td>&lt;.0001</td>
<td>1.7898</td>
<td>4.4946</td>
</tr>
</tbody>
</table>

---

signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Campbell Webinar
2. Approaches to SCD Meta-analysis

The What Works Clearinghouse (WWC) hosted a webinar (presenter Mariola Moeyaert) on October 13, 2020, to discuss the use, application, and estimation of the between-case standardized mean difference (BC-SMD) for single-case design studies (SCDs). For more details, see https://youtu.be/uXTbL8QkNvY.
2. Approaches to SCD Meta-analysis

2.2 Meta-analysis of Case-Level Effect Sizes
2. Approaches to SCD Meta-analysis

2.2 Meta-analysis of Case-Level Effect Sizes

**Effect size:** Single-number summary of intervention effect *for each case.*


24. Grünke et al., 2013

45. Stringfield et al., 2011
2.2 Meta-analysis of Case-Level Effect Sizes

**Goal:** Compare results across participants and SCD studies *that use various outcome measures.*

- Examine heterogeneity of effects within and between studies.
- Examine case-level predictors of effects.
- Available **ES metrics:** many, some appropriate for non-normal outcome distributions.
- **Modeling assumptions:**
  - Are case specific.
  - Most available ES assume no time trends.
  - Standard errors assume no autocorrelation.
2. Approaches to SCD Meta-analysis

2.2 Meta-analysis of Case-Level Effect Sizes

Meta-analysis methods:

• Case-level ES estimates have a *hierarchical structure*.
  − Effect size estimates for each case (sometimes multiple ES per case)
  − Multiple cases nested in each study

• This suggests that ES from the same study will be *dependent*.
  − Can’t use basic meta-analysis model (assumes all ES are independent)

• Two possible approaches to summarizing ES (Chen & Pustejovsky, in press):
  − Simple average/basic linear regression + Cluster-robust variance estimation
  − Multilevel meta-analysis/meta-regression + Cluster-robust variance estimation
2. Approaches to SCD Meta-analysis

2.2 Meta-analysis of Case-Level Effect Sizes

Meta-analysis methods:

- Simple average:  \[ ES_{ij} = \beta + \epsilon_{ij} \]
  - Cluster-robust variance estimation accounts for dependent, heteroskedastic errors.
  - Provides results describing average effects and possible moderators, but not heterogeneity of effects.
  - Estimated using clubSandwich package in R.

- Multilevel meta-analysis:  \[ ES_{ij} = \beta + u_j + v_{ij} + e_{ij} \]
  - Variance components describe heterogeneity of effects across studies (\( \tau^2 \)) and across cases nested within studies (\( \omega^2 \)).
  - Cluster-robust variance estimation to account for dependency, possible misestimation of \( SE_{ij} \) (due to autocorrelation, small sample size, etc.).
  - Estimated using metafor package in R.
2. Approaches to SCD Meta-analysis

2.2 Meta-analysis of Case-Level Effect Sizes

Available effect size metrics:

- Non-overlap indices (e.g., PND, NAP, Tau, PEN, PAND)
- Mean-based indices (e.g., log response ratio, within-case SMD)
- Regression-based effect sizes
2.2 Meta-analysis of Case-Level Effect Sizes

Available effect size metrics:

- **Non-overlap indices**
  (e.g., PND, NAP, Tau, PEN, PAND)

  - Calculation of non-overlap between baseline and successive intervention phases → derivation of a percentage score.
  - Easy to interpret: The higher the percentage, the more effective the treatment.
  - Nonparametric (scores usually not independent and normally distributed).
  - Developed without reference to parametric distributional modeling assumptions → lack known sampling variances (Shadish et al., 2008).
  - Not reflecting magnitude of the effect.
2.2 Meta-analysis of Case-Level Effect Sizes

Available effect size metrics:

• Mean-based indices – log response ratio

\[ R = \ln \left( \frac{\bar{Y}_B}{\bar{Y}_A} \right) \]

- The natural logarithm is used to make distribution easier to work with.
- If the means are equal, then \( R = 0 \).
2. Approaches to SCD Meta-analysis

2.2 Meta-analysis of Case-Level Effect Sizes

Available effect size metrics:

• **Mean-based indices:** Within-case standardized mean difference ($SMD_W$)

\[
SMD_W = \left(1 - \frac{3}{4m - 5}\right) \times \frac{\bar{Y}_B - \bar{Y}_A}{s_A}
\]

- The $SMD_W$ is standardized by the baseline standard deviation ($s_A$) and corrected for small $n$ bias.
2. Approaches to SCD Meta-analysis

2.2 Meta-analysis of Case-Level Effect Sizes

Available effect size metrics:

- Regression-based effect sizes

\[
y = \beta_0 + (\beta_1 \times Trt) + (\beta_2 \times Time) + (\beta_3 \times Trt \times Time) + e
\]

\[b_3 = \Delta \text{slope}\]

\[b_2 = \text{slope}_A\]

\[b_1\]

\[b_0\]
2. Approaches to SCD Meta-analysis

2.2 Meta-analysis of Case-Level Effect Sizes

Meta-analysis method depends on the effect size metric; see Chen & Pustejovsky (in press).

<table>
<thead>
<tr>
<th>Metric</th>
<th>Strategy</th>
<th>Non-normal outcomes</th>
<th>Auto-correlation</th>
<th>Time trends</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log response ratio</td>
<td>Multilevel meta-analysis</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within-case SMD</td>
<td>Simple average</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-overlap of All Pairs</td>
<td>Simple average</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tau(AB)</td>
<td>Simple average</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. Approaches to SCD Meta-analysis

2.2 Meta-analysis of Case-Level Effect Sizes – Illustration

Data organization:

<table>
<thead>
<tr>
<th>Study</th>
<th>Case</th>
<th>ES</th>
<th>Est</th>
<th>SE</th>
<th>CI_lower</th>
<th>CI_upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>NAP</td>
<td>1.000000</td>
<td>0.000000</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>NAP</td>
<td>1.000000</td>
<td>0.000000</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>NAP</td>
<td>0.902140</td>
<td>0.017074</td>
<td>0.876568</td>
<td>0.927712</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>NAP</td>
<td>1.000000</td>
<td>0.000000</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>NAP</td>
<td>0.854167</td>
<td>0.148195</td>
<td>0.664928</td>
<td>1.043406</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>NAP</td>
<td>1.000000</td>
<td>0.000000</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>NAP</td>
<td>1.000000</td>
<td>0.000000</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>NAP</td>
<td>1.000000</td>
<td>0.000000</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
<td>NAP</td>
<td>1.000000</td>
<td>0.000000</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>NAP</td>
<td>1.000000</td>
<td>0.000000</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>11</td>
<td>9</td>
<td>NAP</td>
<td>1.000000</td>
<td>0.000000</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>12</td>
<td>9</td>
<td>NAP</td>
<td>0.965667</td>
<td>0.055519</td>
<td>0.864756</td>
<td>1.066579</td>
</tr>
<tr>
<td>13</td>
<td>24</td>
<td>NAP</td>
<td>1.000000</td>
<td>0.015062</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>14</td>
<td>24</td>
<td>NAP</td>
<td>1.000000</td>
<td>0.010362</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>15</td>
<td>24</td>
<td>NAP</td>
<td>1.000000</td>
<td>0.015062</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>16</td>
<td>24</td>
<td>NAP</td>
<td>1.000000</td>
<td>0.011197</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>17</td>
<td>24</td>
<td>NAP</td>
<td>1.000000</td>
<td>0.012534</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>18</td>
<td>24</td>
<td>NAP</td>
<td>1.000000</td>
<td>0.009929</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
<tr>
<td>19</td>
<td>29</td>
<td>NAP</td>
<td>0.933243</td>
<td>0.007567</td>
<td>0.826163</td>
<td>1.040324</td>
</tr>
<tr>
<td>20</td>
<td>29</td>
<td>NAP</td>
<td>0.871794</td>
<td>0.063231</td>
<td>0.745560</td>
<td>1.000000</td>
</tr>
<tr>
<td>21</td>
<td>29</td>
<td>NAP</td>
<td>0.852000</td>
<td>0.245535</td>
<td>0.308535</td>
<td>1.000000</td>
</tr>
<tr>
<td>22</td>
<td>29</td>
<td>NAP</td>
<td>0.953333</td>
<td>0.023815</td>
<td>0.906458</td>
<td>1.000000</td>
</tr>
<tr>
<td>23</td>
<td>29</td>
<td>NAP</td>
<td>1.000000</td>
<td>0.028625</td>
<td>1.000000</td>
<td>1.000000</td>
</tr>
</tbody>
</table>

Zaias et al., 2015
Zaias et al., 2014
Zaias et al., 2013
Campbell Webinar

0.00  0.25  0.50  0.75  1.00
NAP (increasing)
2. Approaches to SCD Meta-analysis

2.2 Meta-analysis of Case-Level Effect Sizes – Illustration

Output:

NAP = .94, SE = 0.049, \( t(8.99) = 19.36, p < .001 \)

\( \omega^2 = 0.0238 \) and \( \tau^2 = 0.00 \)

- The NAP is very large.
- There is little between-participant and between-study variability in intervention effectiveness.

---

Multivariate Meta-Analysis Model (k = 41; method: REML)

<table>
<thead>
<tr>
<th>loglik</th>
<th>deviance</th>
<th>AIC</th>
<th>BIC</th>
<th>AICC</th>
</tr>
</thead>
<tbody>
<tr>
<td>62.8102</td>
<td>-125.6204</td>
<td>-118.6204</td>
<td>-114.5538</td>
<td>-118.9538</td>
</tr>
</tbody>
</table>

Variance Components:

<table>
<thead>
<tr>
<th>estim</th>
<th>sort</th>
<th>nvlvs</th>
<th>fixed</th>
<th>factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>sigma2.1</td>
<td>0.0238</td>
<td>0.1542</td>
<td>10</td>
<td>no Study</td>
</tr>
<tr>
<td>sigma2.2</td>
<td>0.0000</td>
<td>0.0000</td>
<td>41</td>
<td>no Study/Case</td>
</tr>
</tbody>
</table>

Test for Heterogeneity:

Q(df = 40) = 153.7559, p-val < .0001

Number of estimates: 41
Number of clusters: 10
Estimates per cluster: 3-6 (mean: 4.10, median: 3)

Model Results:

<table>
<thead>
<tr>
<th>estimate</th>
<th>se</th>
<th>tval</th>
<th>df</th>
<th>pval</th>
<th>ci.lb</th>
<th>ci.ub</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9463</td>
<td>0.0489</td>
<td>19.366</td>
<td>8.99</td>
<td>&lt;.0001</td>
<td>0.8337</td>
<td>1.0569</td>
</tr>
</tbody>
</table>

---

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

1) results based on cluster-robust inference (var-cov estimator: CR2, approx t-test and confidence interval, df: Satterthwaite approx)
2. Approaches to SCD Meta-analysis

2.3 Raw Data Meta-analysis
2. Approaches to SCD Meta-analysis

2.3 Raw Data Meta-analysis

- Raw SCD data meta-analysis is also called raw individual patient/participant data (IPD) meta-analysis (Declercq et al., 2020; Moeyaert & Fingerhut, 2022).

- Raw data from multiple participants and studies are synthesized.

- Three-level structure:

  **Level 3:**
  Studies

  **Level 2:**
  Cases

  **Level 1:**
  Measurements
2.3 Raw Data Meta-analysis

Goals:

• Modeling the three-level hierarchical structure
• Estimating the overall intervention effect across cases and across studies in addition to participant-specific and study-specific treatment effects
• Estimating between-participant and between-study variance
• Investigating moderators at both the case and study levels to explain intervention heterogeneity

Source: Moeyaert & Yang (2021)
2. Approaches to SCD Meta-analysis

2.3 Raw Data Meta-analysis

**Meta-analysis method:** IPD meta-analysis

**Level 1:** Variation of scores within participants

\[ y_{ijk} = \beta_{0jk} + \beta_{1jk} \times Trt_{ijk} + e_{ijk} \]

**Level 2:** Variation between participants from the same study

\[ \beta_{0jk} = \theta_{00k} + u_{0jk} \]

\[ \beta_{1jk} = \theta_{10k} + u_{1jk} \]

\[ \begin{pmatrix} u_{0jk} \\ u_{1jk} \end{pmatrix} \sim MVN \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{u0}^2 & \sigma_{u1}^2 \\ \sigma_{u01} & \sigma_{u1}^2 \end{pmatrix} \right) \]

**Level 3:** Variation between studies

\[ \theta_{00k} = \gamma_{000} + v_{00k} \]

\[ \theta_{10k} = \gamma_{100} + v_{10k} \]

\[ \begin{pmatrix} v_{0jk} \\ v_{1jk} \end{pmatrix} \sim MVN \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{v0}^2 & \sigma_{v01} \\ \sigma_{v01} & \sigma_{v1}^2 \end{pmatrix} \right) \]
2. Approaches to SCD Meta-analysis

2.3 Raw Data Meta-analysis

**Meta-analysis method:** IPD meta-analysis

Combined model:

\[
y_{ijk} = \gamma_{00} + u_{0jk} + \nu_{00k} + (\gamma_{10} + u_{1jk} + \nu_{10k}) \times Trt_{ijk} + e_{ijk}
\]

with \( e_{ijk} \sim N(0, \sigma_e^2) \) and

\[
\begin{bmatrix}
u_{0jk} \\
u_{1jk}
\end{bmatrix} \sim \text{MVN}\left[ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{u0}^2 & \sigma_{u01} \\ \sigma_{u01} & \sigma_{u1}^2 \end{bmatrix} \right]
\]

Meta-analysts are interested in the **estimate of** \( \gamma_{10} \), which expresses the overall intervention effect across participants and studies; and in the variance component \( \sigma_{u1}^2 \), which expresses the extent to which the intervention effect varies across participants within the study; and in the variance component \( \sigma_{v1}^2 \), which expresses the extent to which the intervention varies across studies.
2. Approaches to SCD Meta-analysis

2.3 Raw Data Meta-analysis

- Example of two studies included in empirical illustration:

\[ y'_{ijk} = \frac{y_{ijk}}{\hat{\sigma}_{ejk}}, \]

\[ \hat{\sigma}_{ejk} \text{ is obtained by running the following OLS per participant: } y_{ijk} = \beta_{0jk} + \beta_{1jk} \times Trt_{ijk} + e_{ijk} \]

- Therefore, **standardization is needed** (Moeyaert et al., 2013; Van den Noortgate & Onghena, 2008).

- The outcome data \((y_{ijk} s)\) are standardized by dividing them by the estimated residual within-subject standard deviation of participant \(j\) from study \(k\), \(\hat{\sigma}_{ejk}\) (Van den Noortgate & Onghena, 2008):
2. Approaches to SCD Meta-analysis

2.3 Raw Data Meta-analysis

The major advantage of using IPD meta-analysis is its flexibility.

Variety of modeling options:
• Heterogeneity of variances
• Autocorrelation
• Count outcomes
• Nonlinear time trend
• Multiple moderators

The major limitation is modeling assumptions:
• Mis-specification of data trends
• Normality of level 1, level 2, and level 3 residuals
2. Approaches to SCD Meta-analysis

2.3 Raw Data Meta-analysis – Illustration

Data organization:
2. Approaches to SCD Meta-analysis

2.3 Raw Data Meta-analysis – Illustration

Results of the raw data meta-analysis (i.e., three-level hierarchical linear modeling):

\[ \gamma_{100} = 8.26, \ SE = 4.14, t(719) = 1.99, p = .0461 \]
\[ \sigma_{u_1}^2 = 25.11^2 \text{ and } \sigma_{v_1}^2 = 4.11^2 \]

- There is a substantial amount of between-participant and between-study variability in intervention effectiveness.
- Most variability is between participants.
3. Concluding Thoughts

- Currently, the three different SCD meta-analytic approaches are not well understood/disseminated.

- The appropriate meta-analytic approach depends on the research question, level of analysis, available effect metrics, modeling assumptions.

- Both applied SCD researchers and methodologists need to work on clarifying research questions, level of analysis, available effect metrics, and modeling assumptions. Through their efforts, the most appropriate approach can be rationalized.
References


References (cont.)


References (cont.)


Additional Resources

Evidence-Based Communication Assessment and Intervention, Vol. 14, 1–2 & Vol. 17, 1–2

#1 – Advances in Statistical Analysis and Meta-analysis of Single-Case Experimental Designs
#2 – New Developments in Meta-analysis of Single-Case Experimental Designs

https://www.tandfonline.com/journals/tebc20/special-issues

Selecting the proper Tau-U measure for single-case experimental designs: Development and application of a decision flowchart

Joelle Fingerhut, Xinyun Xu & Mariola Moeyaert

Effect size estimation for combined single-case experimental designs

Mariola Moeyaert, Diana Akhmedjanova, John Ferron, S. Natasha Beretvas & Wim Van den Noortgate
Follow-up Webinar

Fall 2023:


Speakers:

Mariola Moeyaert, Ralf Schlosser, & Oliver Wendt

Campbell Webinar Series 2023

Questions?

Mariola Moeyaert, PhD, MBA
mmoeyaert@albany.edu
Associate Professor of Applied Statistics
University at Albany – SUNY
February 7, 2023
The contents of this presentation were developed under grant number 90DPKT0010 from the National Institute on Disability, Independent Living, and Rehabilitation Research (NIDILRR). NIDILRR is a Center within the Administration for Community Living (ACL), Department of Health and Human Services (HHS). The contents of this presentation do not necessarily represent the policy of NIDILRR, ACL, HHS, and you should not assume endorsement by the Federal Government.

Evaluation
Please let us know what you thought about this webcast!